Lecture 9

Proof of $p \Rightarrow q$ part

(i.e. $L = L(N)$ ... $\Rightarrow$ $L = L(\alpha)$ ...)

NFA-to-RE?

To prove this, we need to define a new comp. model: "Generalized NFA" (GNFA)

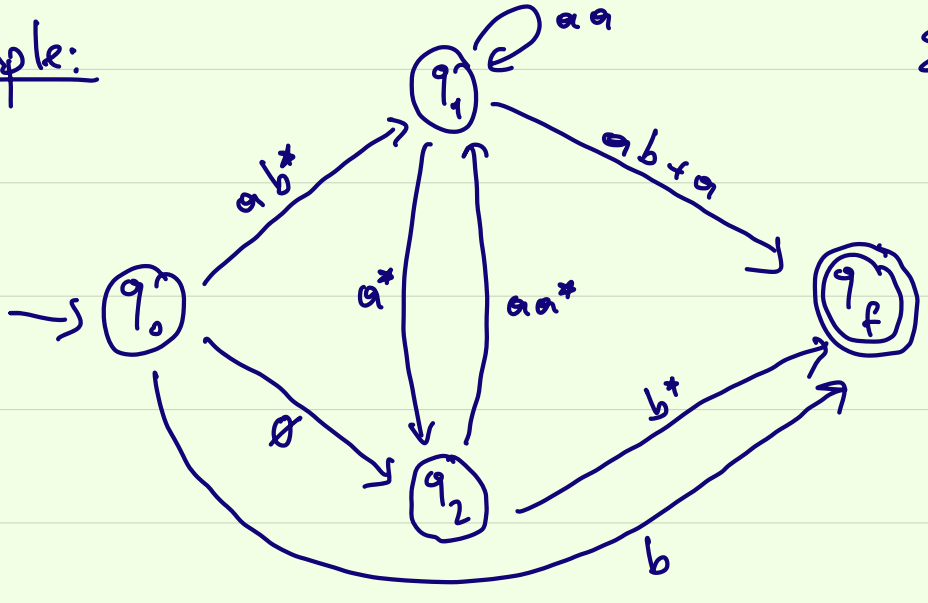
A GNFA has

- a unique init. state q_0
- a " final state q_f
- no transitions going out of q_f
- no transitions coming into q_0
- transitions labelled with reg. exp.

Example:

$\Sigma = \{a, b\}$

N



- $w = b \in L(N)$
- $w = a \in L(N)$

$$q_0 \xrightarrow{a} q_1 \xrightarrow{\varepsilon} q_2 \xrightarrow{\varepsilon} q_f$$

- $w = a^2 \in L(N)$

$$q_0 \xrightarrow{a} q_1 \xrightarrow{a} q_2 \xrightarrow{\varepsilon} q_f$$

- $w = ab^3a^4b \in L(N)$

$$q_0 \xrightarrow{ab^3} q_1 \xrightarrow{a^4} q_2 \xrightarrow{b} q_f$$

- $w = ab^3a^4b^2 \in L(N)$
- $w = ba \notin L(N)$
- $w = \varepsilon$

A GNFA N is defined as a 5-tuple

$N = (Q, \Sigma, \delta, q_0, q_f)$ where

$$\delta : Q - \{q_f\} \times Q - \{q_0\} \longrightarrow RE$$

$$(q) \xrightarrow{RE} (q')$$

(i.e. $\delta : Q - \{q_f\} \times RE \to Q - \{q_0\}$)

Definition: A string $w = x_1 \cdot x_2 \cdots x_n$

(x_i, $i = 1, \ldots, n$, are substrings of w)

is accepted by a GNFA $N = (Q, \Sigma, \delta, q_0, q_f)$

if there exists a sequence of states

$r_0, r_1, \ldots, r_n$ such that

i) r_0 is init. state, i.e. $r_0 = q_0$

ii) r_n is final state, i.e., $r_n = q_f$

iii) $\delta(r_{i-1}, r_i) = R_i$ and $x_i \in L(R_i)$

Back to proof:

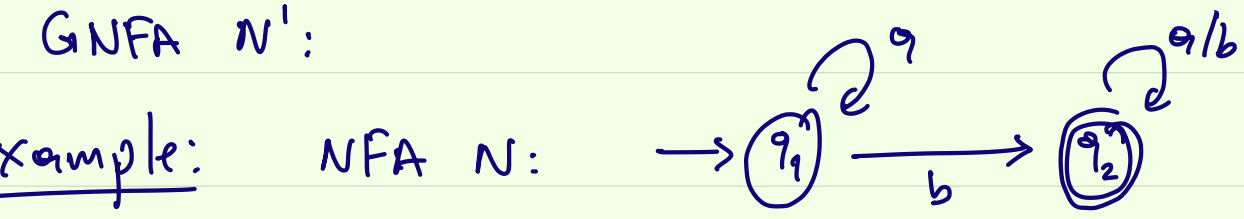
Outline:

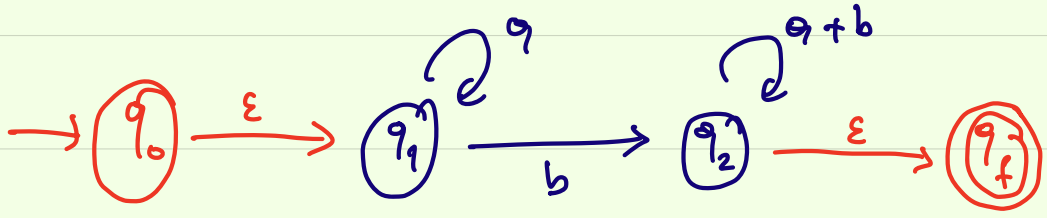
NFA N $\xrightarrow{\text{convert}}$ GNFA N' $\xrightarrow[\text{states}]{\text{Eliminate}}$ GNFA N'' $\longrightarrow$ The equiv. R.E. α

$$(L(N) = L(N') = L(N'') = L(\alpha))$$

1) Convert an NFA N into an equivalent GNFA N':

Example: NFA N:





In general, we

- add q_0 and an ε-transition from it to the old init. state.
- add q_f and ε-transitions from old final states to it.
- Convert every label into a regular expression.

e.g. $O \xrightarrow{a,b} O \longrightarrow O \xrightarrow{a+b} O$

2) Eliminate states from GNFA N' to obtain GNFA N' (with two states only).

The Algorithm:

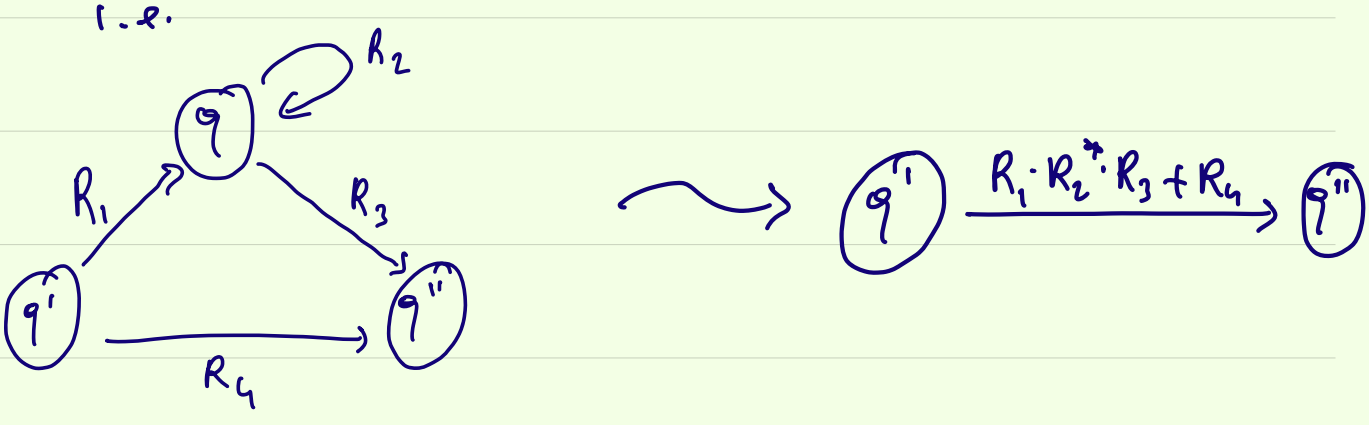
Repeat the following until there are two states left (q_0 and q_f namely).

- Select a state $q \in Q - \{q_0, q_f\}$
- For every pair of states (q', q'') s.t. $q' \in Q - \{q, q_f\}$ and $q'' \in Q - \{q, q_0\}$

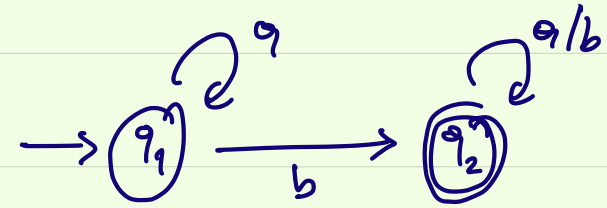
label the transition from q' to q'' with $R_1 \cdot R_2^* \cdot R_3 + R_4$ where

- R_1 is the transition of $q' \to q$
- R_2 " " " " $q \to q$
- R_3 " " " " $q \to q''$
- R_4 " " " " $q' \to q''$

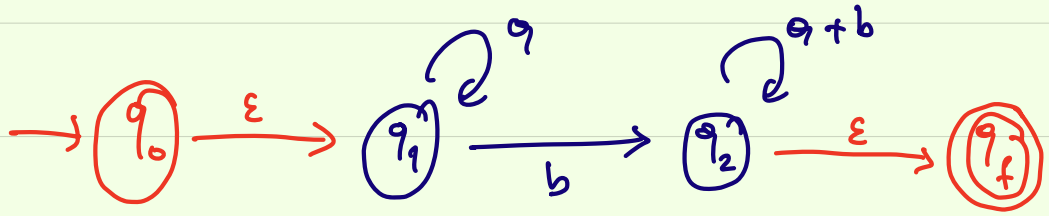
i.e.



Example: NFA N:

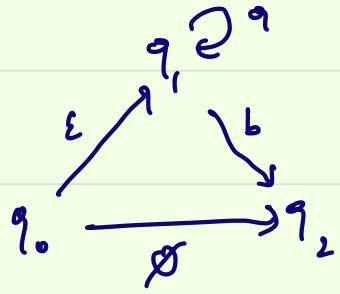


N':



Recall: $\alpha \cdot \varepsilon = \alpha$, $\varepsilon \cdot \alpha = \alpha$, $\alpha + \emptyset = \alpha$, $\alpha \cdot \emptyset \neq \alpha$, $L(\emptyset^*) = \{\varepsilon\}$, $\emptyset^* = \varepsilon$

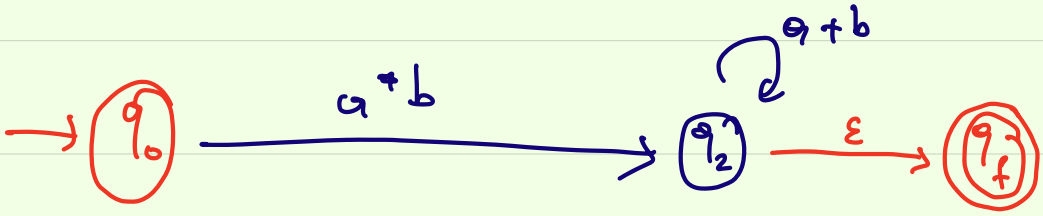
Eliminate q_1:



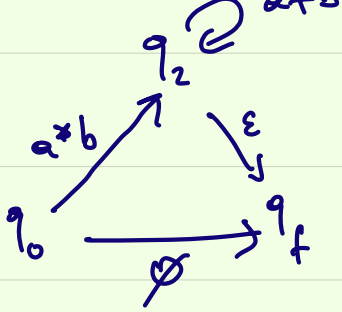
$\rightsquigarrow$ $q_0 \xrightarrow{\varepsilon \cdot a^* b + \emptyset} q_2$

i.e. $q_0 \xrightarrow{a^* b} q_2$

We get,

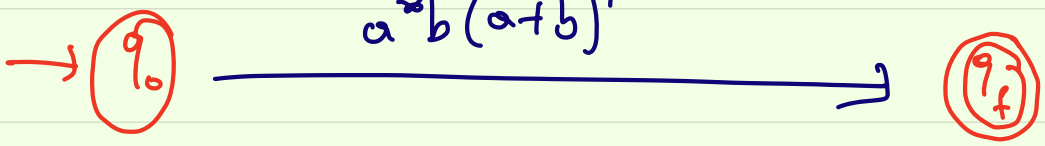


Eliminate q_2:



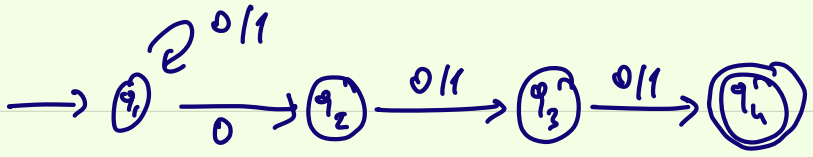
$\rightsquigarrow$ $q_0 \xrightarrow{a^* b (a+b)^* \cdot \varepsilon + \emptyset} q_f$

We get

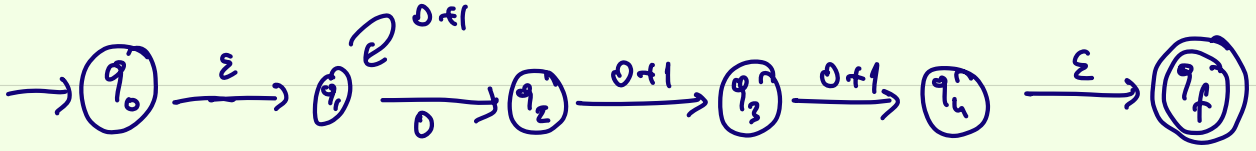


The equiv. RE is $a^*b(a+b)^*$.

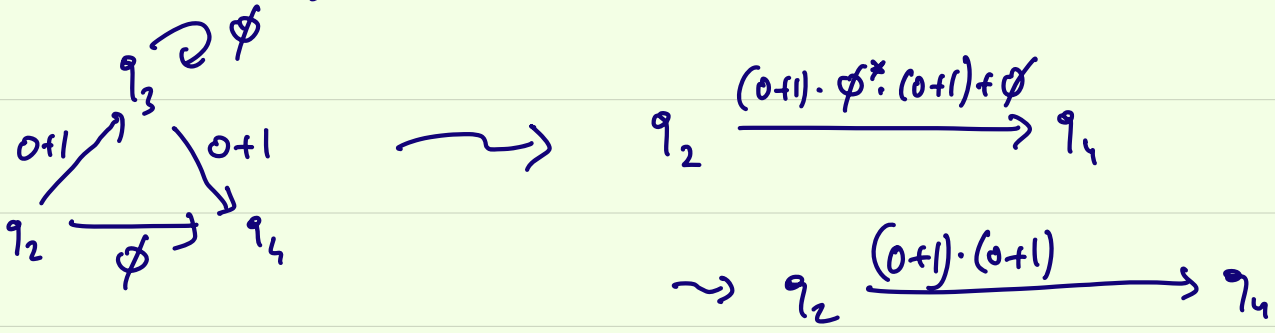
Example: N:

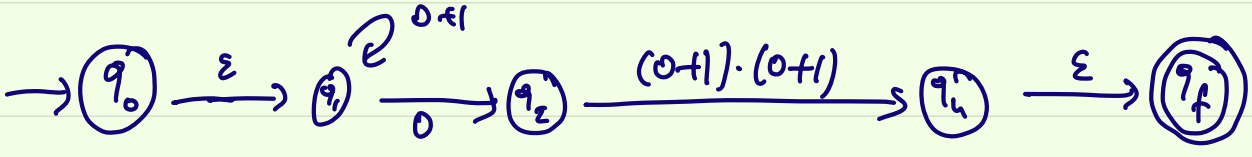


GNFA N':

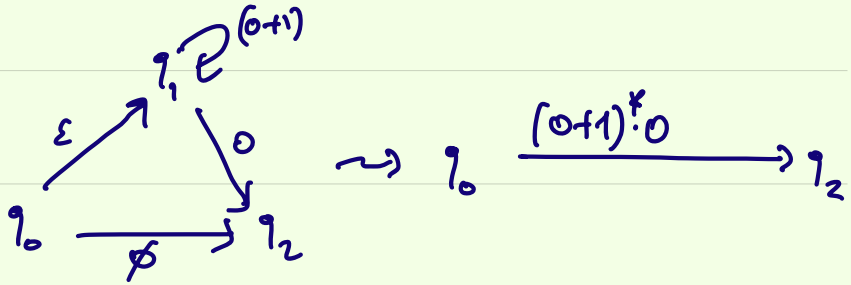


Eliminate q_3:

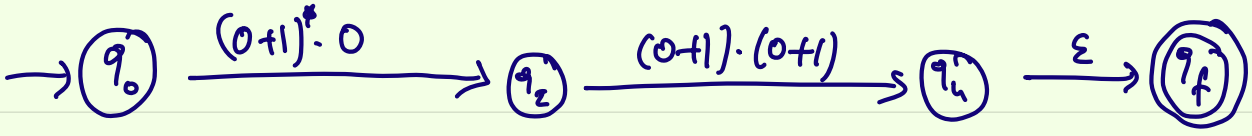




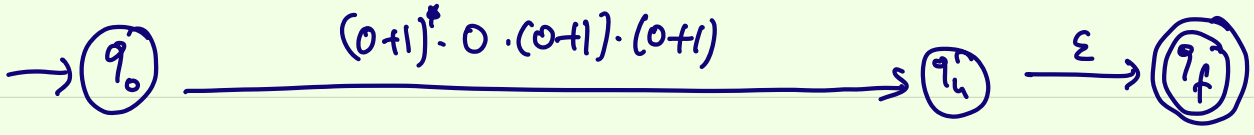
Eliminate q_1:



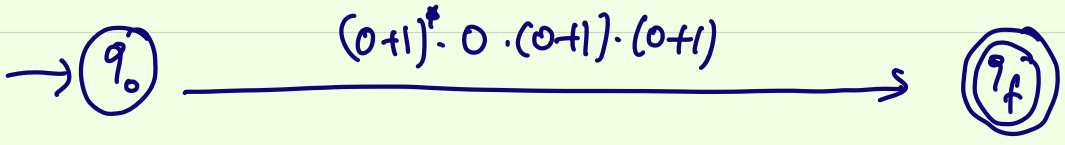
we get



Eliminate q_2:



Eliminate q_4:



The equiv. RE $(0+1)^*\cdot 0\cdot(0+1)\cdot(0+1)$.

Exercise: The equiv. RE for the following NFA N:

