Lecture 8

Equivalence of NFAs and REs

Theorem: $L = L(N)$ for some NFA N (p)

if and only if

$L = L(\alpha)$ for some RE α. (q)

Proof of (q $\Rightarrow$ p):

(i.e. $L = L(\alpha)$ ... $\Rightarrow$ $L = L(N)$ ...)

Idea: Given a reg. exp α, we construct an equiv. NFA N. The construction is defined recursively.

Basis:

- $\alpha = \phi$ ... N: 
- $\alpha = \varepsilon$ ... N: 
- $\alpha = a \in \Sigma$ ... N: 

Rec. Step:

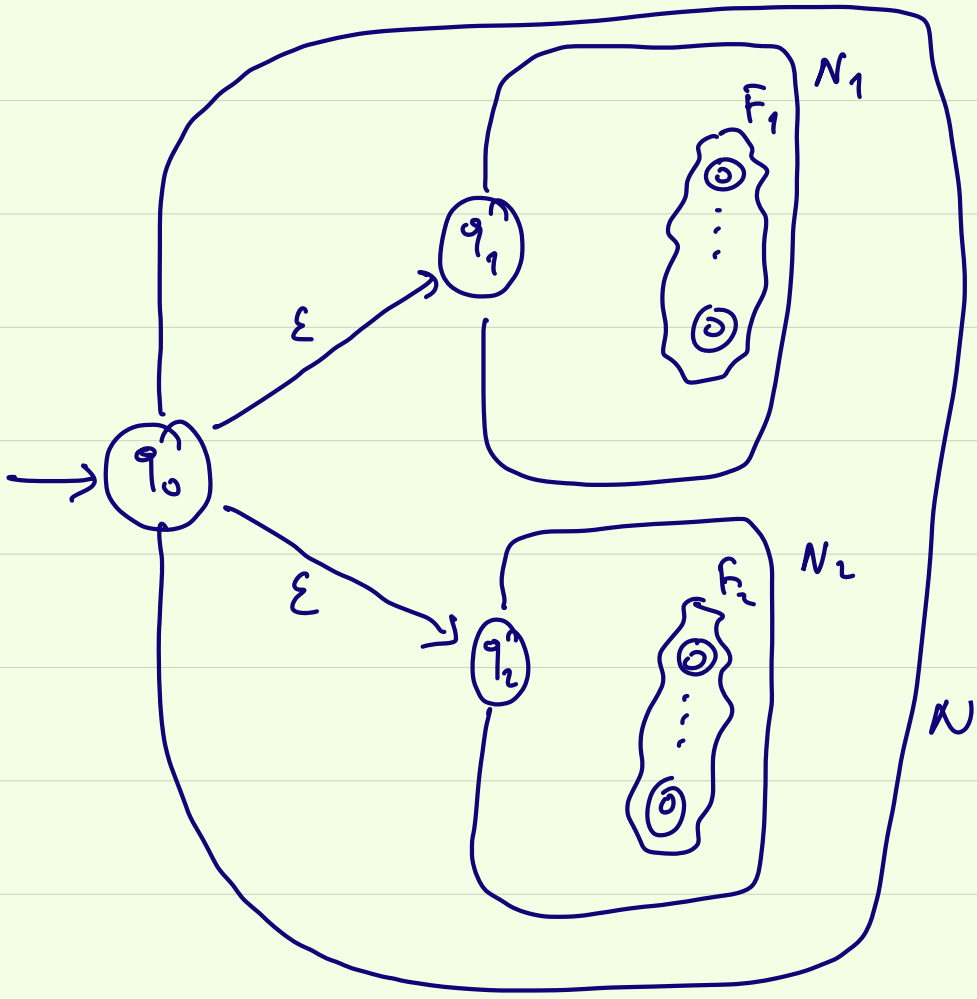
1) $\alpha = \alpha_1 + \alpha_2$ $\qquad L(\alpha) = L(\alpha_1) \cup L(\alpha_2)$

α_1 .. $N_1 = (Q_1, \Sigma, \delta_1, q_1, F_1)$

α_2 .. $N_2 = (Q_2, \Sigma, \delta_2, q_2, F_2)$

α N?

$w \in L(\alpha) \iff w \in L(\alpha_1) \vee w \in L(\alpha_2)$



$$N = (Q_1 \cup Q_2 \cup \{q_0\}, \Sigma, \delta, q_0, F_1 \cup F_2)$$

where

$$\delta(q, a) = \begin{cases} \{q_1, q_2\} & \text{if } q = q_0,\ a = \varepsilon \\ \delta_1(q, a) & \text{if } q \in Q_1 \\ \delta_2(q, a) & \text{if } q \in Q_2 \end{cases}$$

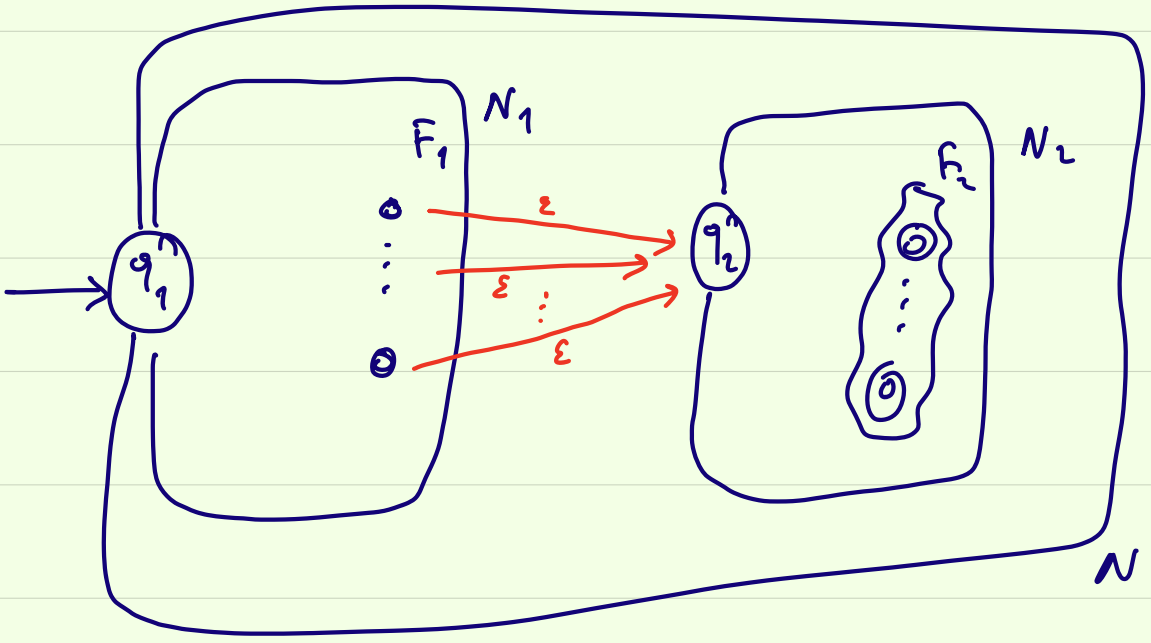
2) $\alpha = \alpha_1 \cdot \alpha_2$ $\qquad L(\alpha) = L(\alpha_1) \cdot L(\alpha_2)$

α_1 .. $N_1 = (Q_1, \Sigma, \delta_1, q_1, F_1)$

α_2 .. $N_2 = (Q_2, \Sigma, \delta_2, q_2, F_2)$

$$L(\alpha) = L(\alpha_1) \cdot L(\alpha_2)$$

$$= \{ x \cdot y \mid x \in L(\alpha_1) \text{ and } y \in L(\alpha_2) \}$$



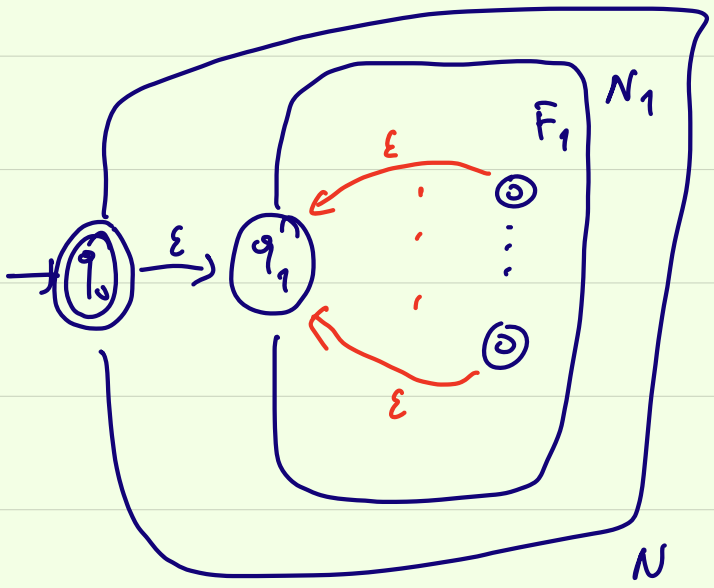
$$N = (Q_1 \cup Q_2, \Sigma, \delta, q_1, F_2)$$

where

$$\delta(q, a) = \begin{cases} \delta_1(q, a) & \text{if } q \in Q_1 - F \\ \delta_1(q, a) & \text{if } q \in F_1, a \neq \varepsilon \\ \delta_1(q, a) \cup \{q_2\} & \text{if } q \in F_1, a = \varepsilon \\ \delta_2(q, a) & \text{if } q \in Q_2 \end{cases}$$

3) $\alpha = \alpha_1^*$ $\qquad L(\alpha) = \left(L(\alpha_1)\right)^* = L(\alpha_1)^0 \cup L(\alpha) \cup \dots$

$\alpha_1 \dots \quad N_1 = (Q_1, \Sigma, \delta_1, q_{1_1}, F_1)$



$w \in L(\alpha)$

$\Rightarrow w \in \left(L(\alpha_1)\right)^*$

$\Rightarrow w = x^n$

$n = 0, 1, \dots$

$N = (Q_1 \cup \{q_0\}, \Sigma, \delta, q_0, F_1 \cup \{q_0\})$

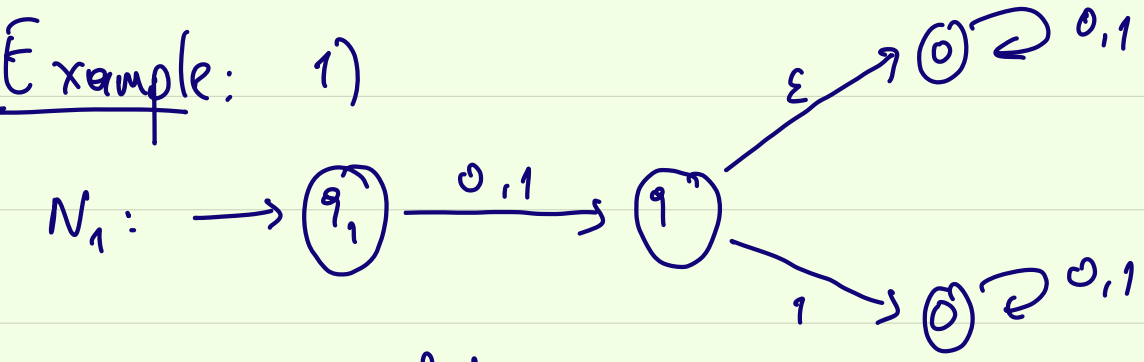
where

$$\delta(q,a) = \begin{cases} \delta_1(q,a) & \text{if } q \in Q_1 - F_1 \\ \delta_1(q,a) & \text{if } q \in F_1,\ a \neq \varepsilon \\ \delta_1(q,a) \cup \{q_1\} & \text{if } q \in F_1,\ a = \varepsilon \\ \{q_1\} & \text{if } q = q_0,\ a = \varepsilon \\ \emptyset & \text{if } q = q_0,\ a \neq \varepsilon \end{cases}$$

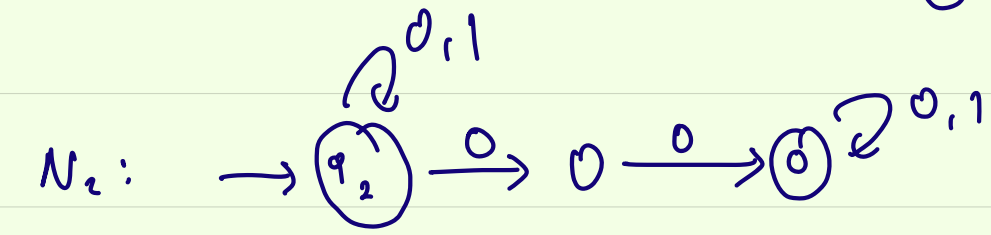
– End of proof of $q \Rightarrow p$ –

Example: 1)

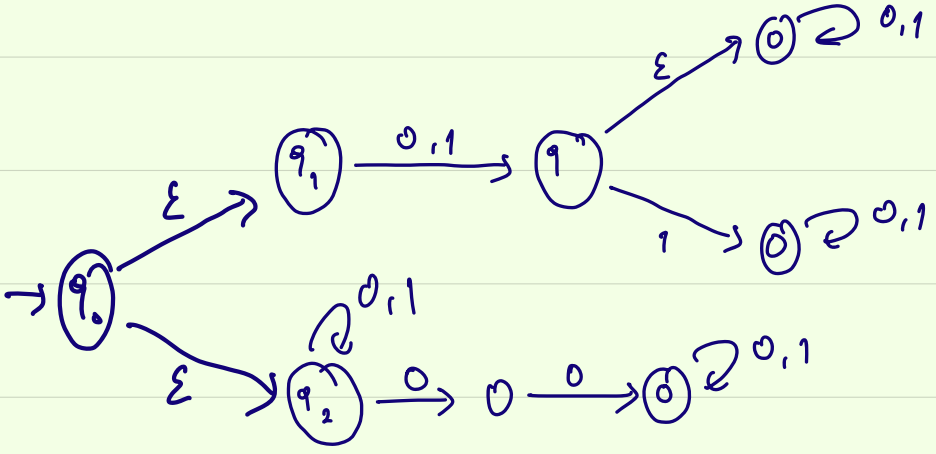
α_1 N_1:



α_2 N_2:

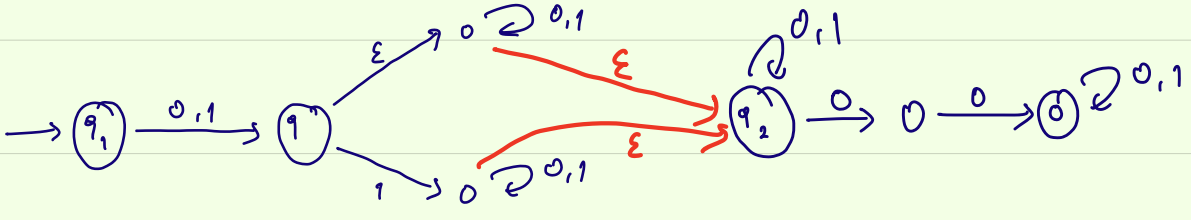


NFA for $\alpha = \alpha_1 + \alpha_2$

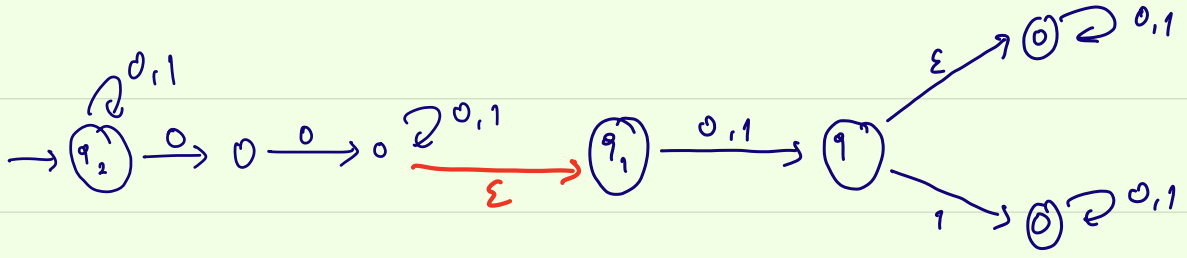


2) NFA for $\alpha = \alpha_2 + \alpha_1$? Same.

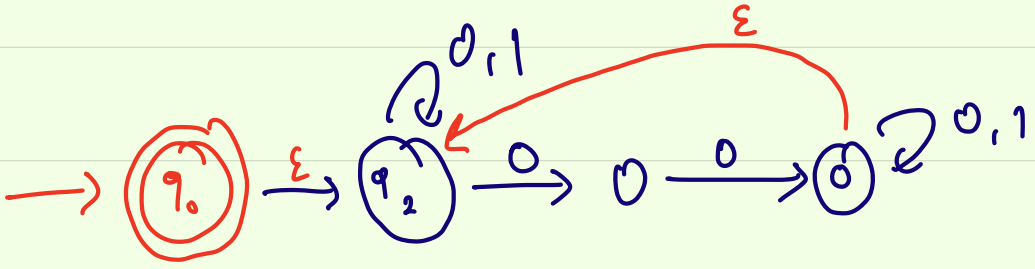
3) NFA $\alpha = \alpha_1 \cdot \alpha_2$



4) NFA $\alpha = \alpha_2 \cdot \alpha_1$



5) NFA for $\alpha = \alpha_2^*$



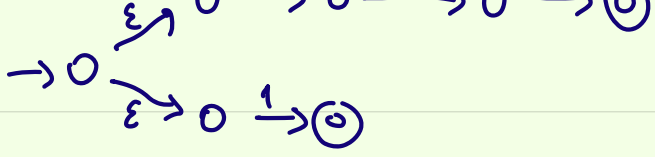
6) $\alpha = (00+1)^*0 + \varepsilon$ $\quad \Sigma = \{0,1\}$

For 0 : $\rightarrow 0 \xrightarrow{0} ((0))$

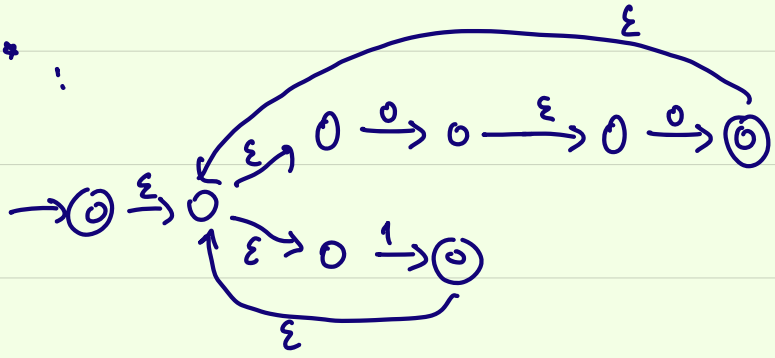
For 00: $\rightarrow 0 \xrightarrow{0} 0 \xrightarrow{\varepsilon} 0 \xrightarrow{0} ((0))$

For 1 : $\rightarrow 0 \xrightarrow{1} ((0))$

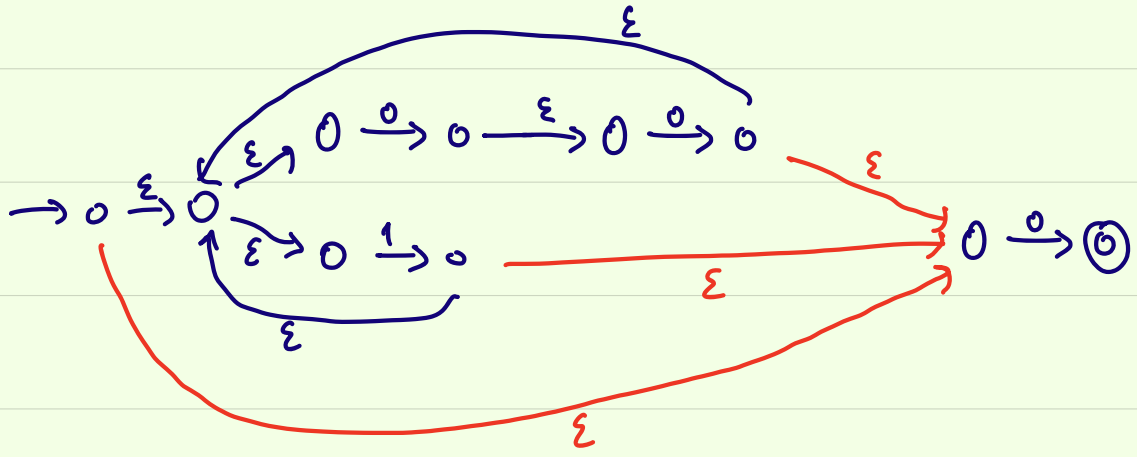
For 00+1 :



For $(00+1)^*$:



For $(00+1)^*.0$



For ε: $\rightarrow 0 \xrightarrow{\varepsilon} ◎$

For $(00+1)^*0 + \varepsilon$:

