Lecture 7

Proof of equivalence of NFAs and DFAs ... contin.

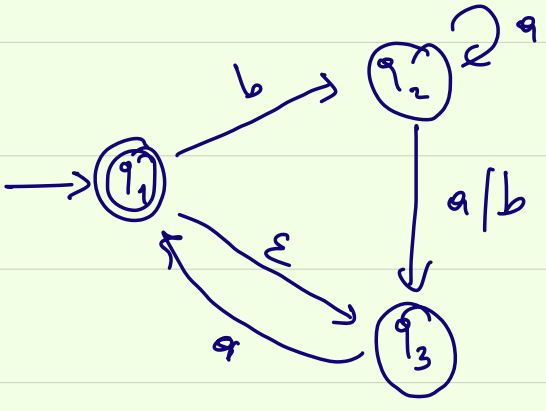
Q: What if an NFA has ε-transitions?

A: Modify the transition function δ' with ε-closures?

Definition: Let $S \in Q'$. Then the ε-closure of S is defined as

$$E(S) = \{ q \in Q \mid q \text{ is reachable from a state in } S \text{ using zero or more } \varepsilon\text{-transitions only} \}$$

e.g.



$S = \{q_1\}$

$\Rightarrow E(S) = \{q_1, q_3\}$

$S = \{q_1, q_2\}$

$\Rightarrow E(S) = \{q_1, q_3, q_2\}$

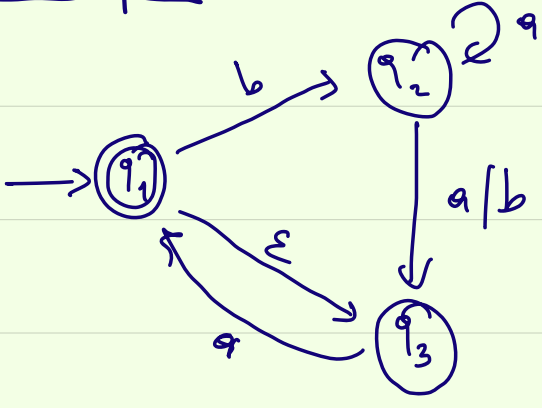
$S = \emptyset \Rightarrow E(S) = \emptyset$.

Then we modify the construction of the equiv. DFA as follows:

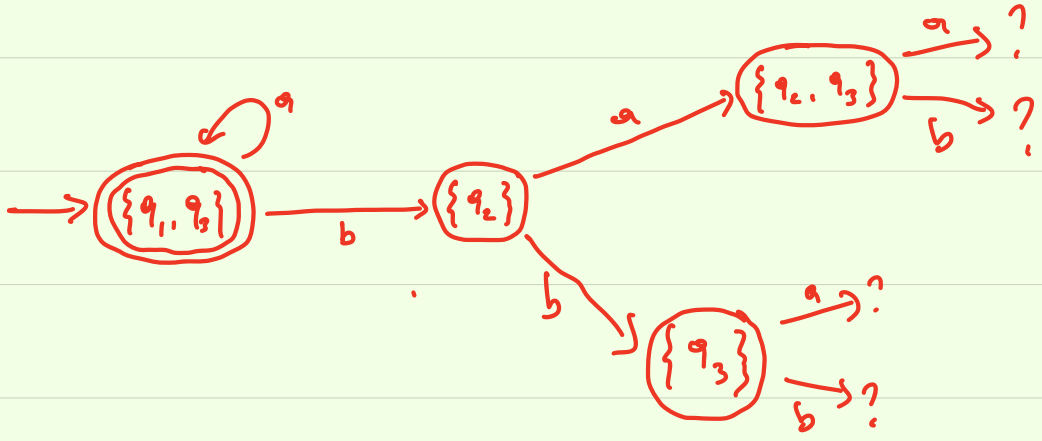
- $Q' \subseteq 2^Q$
- $q_0'' = E(\{q_0\})$
- $F' = \{s \in Q' \mid s \cap F \neq \emptyset\}$
- $\delta''(S, a) = \bigcup_{q \in S} E(\delta(q, a))$

—— ∞ End of Proof of Equivalence ——

Example:



$q_0'' = E(\{q_0\}) = \{q_1, q_3\}$



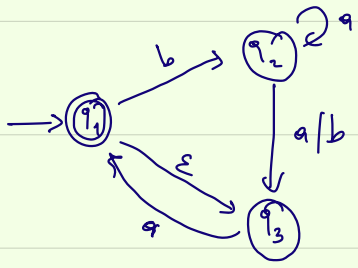
Exercise: Complete the construction.

- $\delta''(\{q_1, q_3\}, a) = E(\delta(q_1, a)) \cup E(\delta(q_3, a))$

$= E(\emptyset) \cup E(\{q_1\})$

$= \emptyset \cup \{q_1, q_3\}$

$= \{q_1, q_3\}$



- $\delta''(\{q_1, q_3\}, b) = E(\delta(q_1, b)) \cup E(\delta(q_3, b))$

$= E(\{q_2\}) \cup E(\emptyset)$

$= \{q_2\}$

- $\delta''(\{q_2\}, a) = E(\delta(q_2, a))$

$= E(\{q_2, q_3\})$

$= \{q_2, q_3\}$

- $\delta''(\{q_2\}, b) = E(\delta(q_2, b))$

$= E(\{q_3\}) = \{q_3\}$

Corollary: A language $L \subseteq \Sigma^*$ is regular if $L = L(N)$ for some NFA N.

REGULAR EXPRESSIONS

Notation: For a reg. exp. α, we write $L(\alpha)$ to denote its language.

Definition: Let Σ be an alphabet. Then the set of all regular expressions over Σ is "recursively" defined as follows:

Basis:
- $\emptyset$ is a reg. exp. $\quad L(\emptyset) = \emptyset$
- λ is a reg. exp. $\quad L(\lambda) = \{\lambda\}$
- $\forall a \in \Sigma$ is a reg. exp. $\quad L(a) = \{a\}$

Rec. Step: If α and β are reg. exp., then
- $\alpha + \beta$ is a reg. exp. $\quad L(\alpha+\beta) = L(\alpha) \cup L(\beta)$
- $\alpha \cdot \beta$ " " $\quad L(\alpha \cdot \beta) = L(\alpha) \cdot L(\beta)$
- α^* " " $\quad L(\alpha^*) = (L(\alpha))^*$

$$\{0\}^* = \{0\}^0 \cup \{0\} \cup \{0\}^2 \cdots$$

Example: $\Sigma = \{0,1\}$

1) $\alpha = 0^* 1 0^*$ $\quad L(\alpha) = ?$

$0 \notin L(\alpha)$ $\quad 011 \in L(\alpha)$

$1 \in L(\alpha)$

$01 \in L(\alpha)$

$10 \in L(\alpha)$

$$L(\alpha) = L(0^* \cdot 1 \cdot 0^*)$$

$$= L(0^*) \cdot L(1) \cdot L(0^*)$$

$$= (L(0))^* \cdot \{1\} \cdot (L(0))^*$$

$$= \{0\}^* \cdot \{1\} \cdot \{0\}^*$$

= the set of all strings with exactly one 1.

$$= \{ w \in \{0,1\}^* \mid |w|_1 = 1 \}$$

2) $\alpha = (0+1)^* 1 (0+1)^*$ $\quad 1 \checkmark$ $\quad 11 \checkmark$

$$L(\alpha) = L((0+1)^* \cdot 1 \cdot (0+1)^*)$$

$$= L((0+1)^*) \cdot L(1) \cdot L((0+1)^*)$$

$$= (L(0+1))^* \cdot L(1) \cdot (L(0+1))^*$$

$$= (L(0) \cup L(1))^* \cdot \{1\} \cdot (L(0) \cup L(1))^*$$

$$= (\{0\} \cup \{1\})^* \cdot \{1\} \cdot (\{0\} \cup \{1\})^*$$

$$= \{0,1\}^* \cdot \{1\} \cdot \{0,1\}^*$$

$$= \Sigma^* \cdot \{1\} \cdot \Sigma^*$$

$$= \{ w \in \{0,1\}^* \mid w = x\,1\,y \quad , x, y \in \Sigma^* \}$$

$$= \{ w \in \{0,1\}^* \mid |w|_1 \geq 1. \}$$

3) $\alpha = (0+1)^*\,010\,(0+1)^*$

$L(\alpha) = \Sigma^* \cdot \{010\} \cdot \Sigma^*$

4) $\alpha = 00\,(0+1)^*\,11$

$L(\alpha) = \{00\} \cdot \Sigma^* \cdot \{11\}$

5) $\alpha = ((0+1)(0+1))^*$

$L(\alpha) = \{ w \in \Sigma^* \mid |w| = 2k \quad , k \in \mathbb{N} \}$

Exercise: Reg. exp. for the set of all odd length strings?