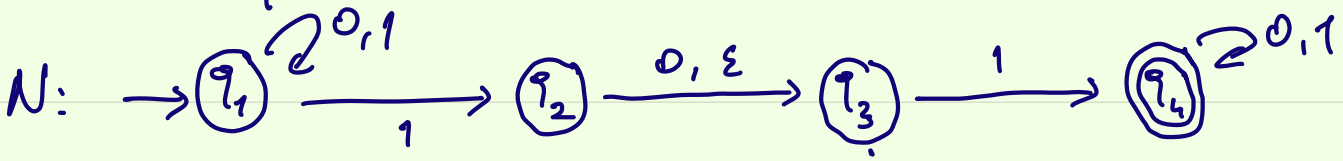
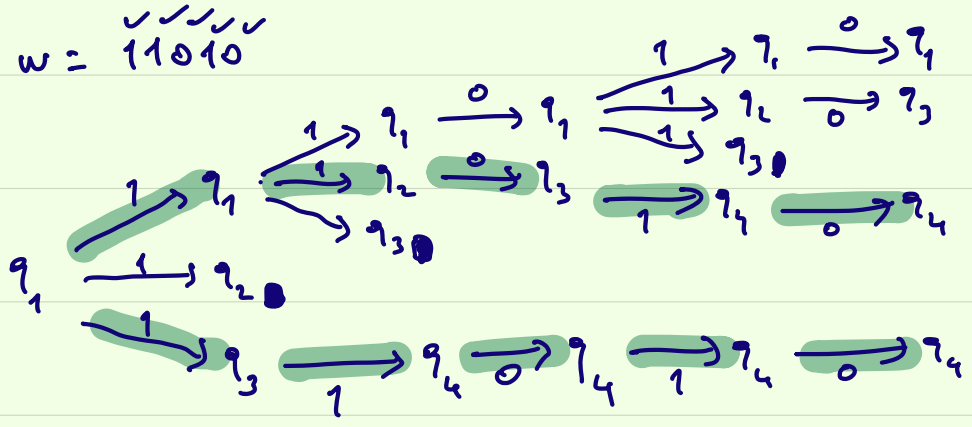
Lecture 6

Example:



$$\delta(q_1, 1) = \{q_1, q_2\}$$
$$\delta(q_1, 0) = \{q_1\}$$
$$\delta(q_3, 0) = \emptyset$$



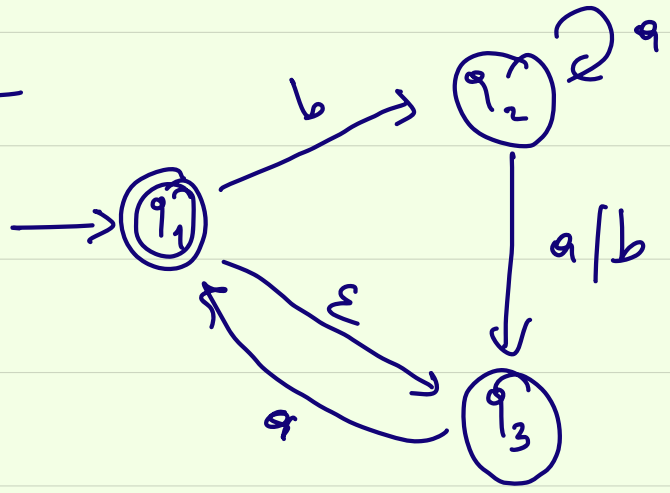
Exercise: Try 001101?

Remark:

In a DFA we have only one sequence of states (i.e. one possible set of state transitions). Yet, in an NFA, there may be more than possible sequence of transitions for a given input.

Example:

N:



$\delta(q_1, \varepsilon) = \{q_3\}$

$\delta(q_2, a) = \{q_2, q_3\}$

δ	a	b	ε
q_1	$\emptyset$	$\{q_2\}$	$\{q_3\}$
q_2	$\{q_2, q_3\}$	$\{q_3\}$	$\emptyset$
q_3	$\{q_1\}$	$\emptyset$	$\emptyset$

$\Sigma = \{a, b\}$

$\Sigma_\varepsilon = \{a, b, \varepsilon\}$

Definition: An NFA N is a 5-tuple

$$N = (Q, \Sigma, \delta, q_0, F)$$

- Q: states
- Σ: input alphabet
- δ: transition function
- q_0: init. state
- F: set of final states

- $\delta : Q \times \Sigma_\varepsilon \to 2^Q$

- $F \subseteq Q$

$\Sigma_\varepsilon = \Sigma \cup \{\varepsilon\}$

2^Q: set of all subsets of Q.

Definition: An input string $w = w_1 w_2 \ldots w_n \in \Sigma^*$ is accepted by an NFA N if there exists a sequence of states $r_0, r_1, \ldots, r_n$ s.t.

- $r_0 = q_0$ (i.e. r_0 should be the init. state.)
- $r_n \in F$ (i.e. r_n should be a final state)
- $r_{i+1} \in \delta(r_i, w_{i+1})$

Theorem: Let $L \subseteq \Sigma^*$ be a language.

Then $L = L(M)$ for some DFA M p

if and only if

$L = L(N)$ for some NFA N. q

To prove $p \iff q$ we need to prove

- $p \Rightarrow q$
- $q \Rightarrow p$

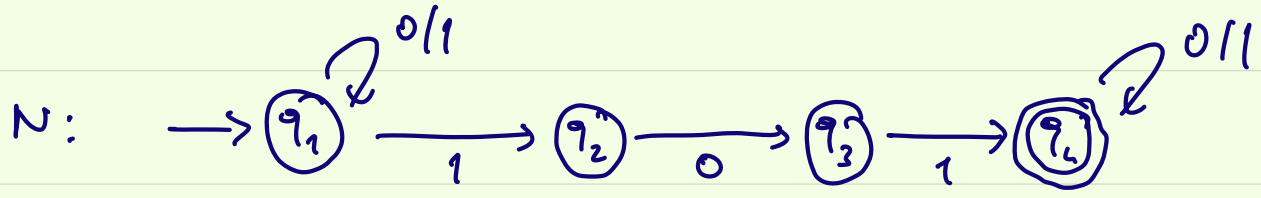
Proof of $p \Rightarrow q$: ($L = L(M) \ldots \Rightarrow L = L(N) \ldots$)

Trivial. Note that every DFA M is an NFA by defn.

Proof of $q \Rightarrow p$: ($L = L(N) \ldots \Rightarrow L = L(M) \ldots$)

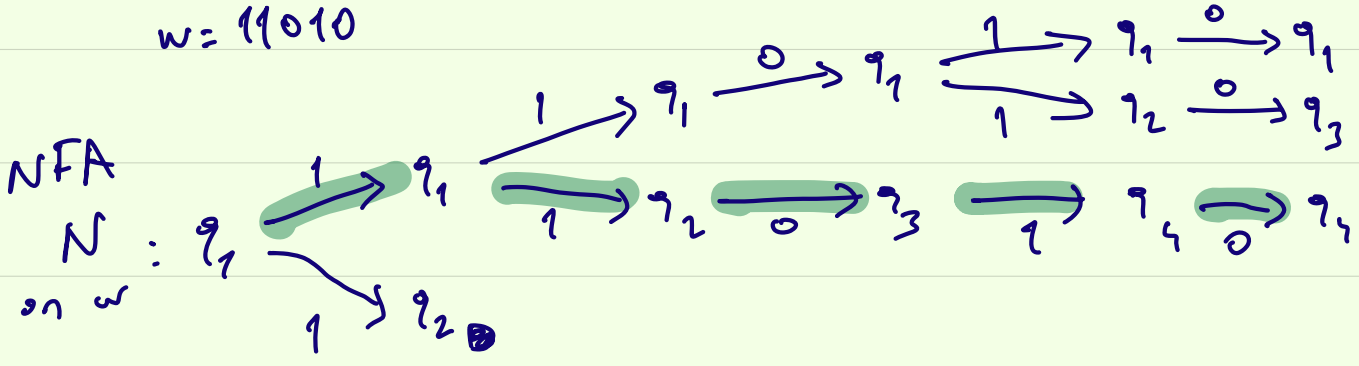
Idea: Given an NFA $N = (Q, \Sigma, \delta, q_0, F)$, construct a DFA $M = (Q', \Sigma, \delta', q_0', F')$ s.t. $L(N) = L(M)$.

For example, take the following NFA N:

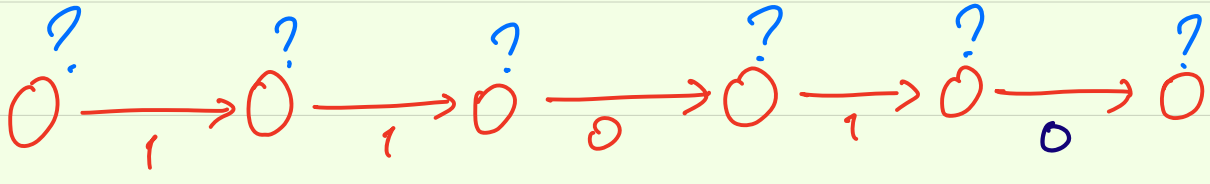


$w = 11010$

NFA N on w:

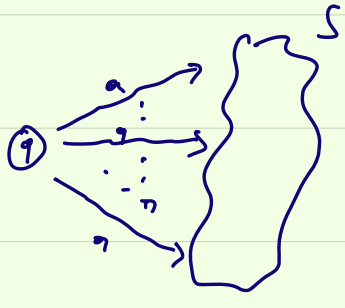


The equiv. DFA on w:

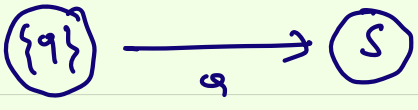


IDEA:

NFA:

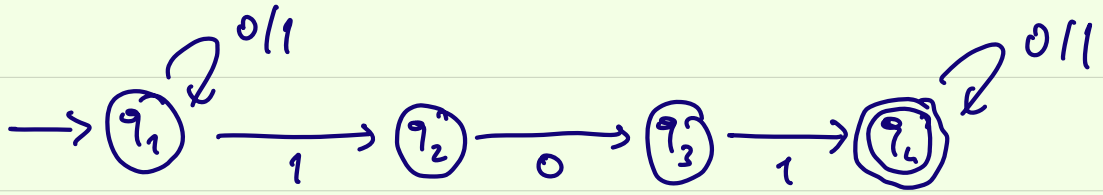


DFA:

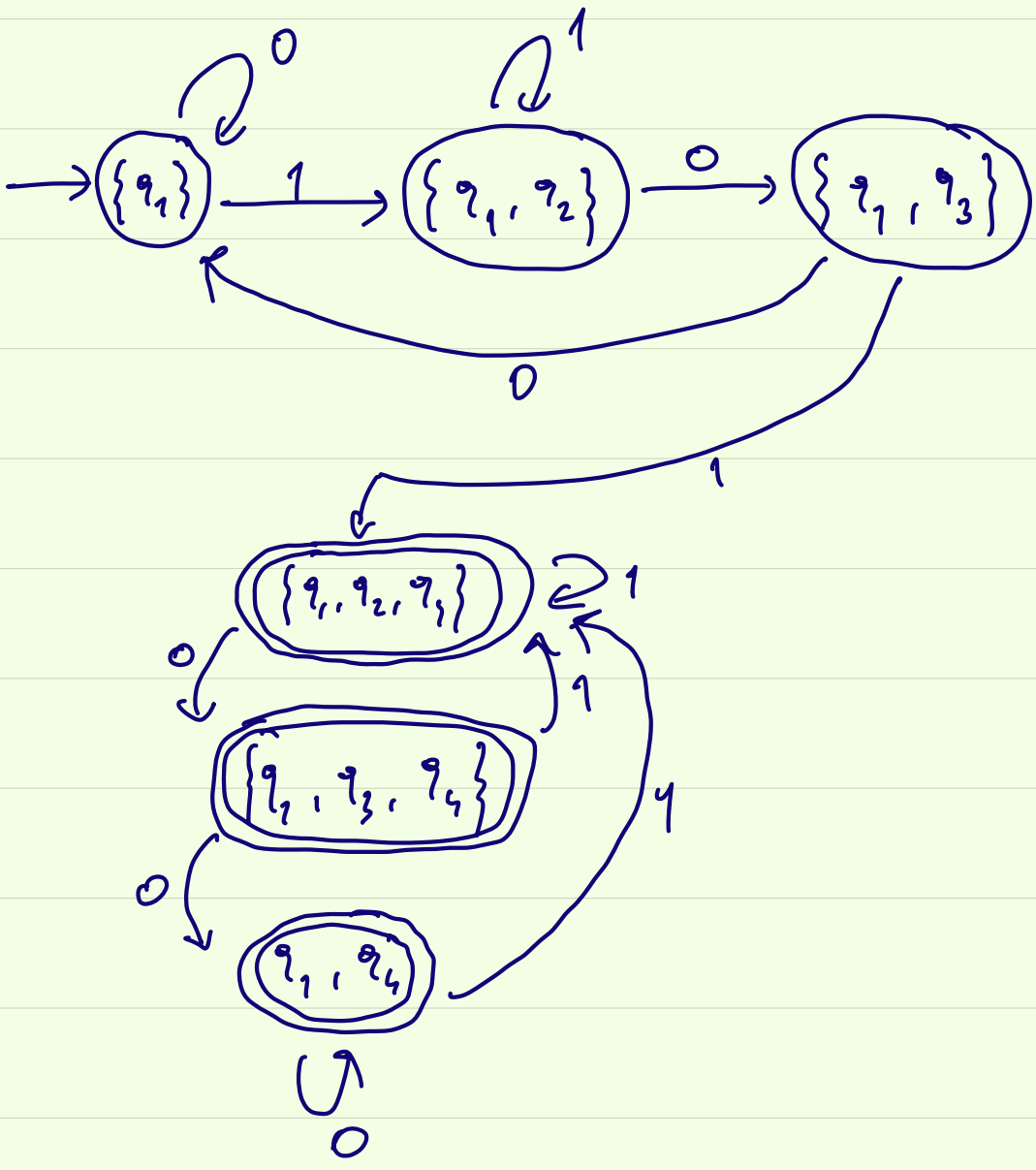


"subset construction"

NFA N:

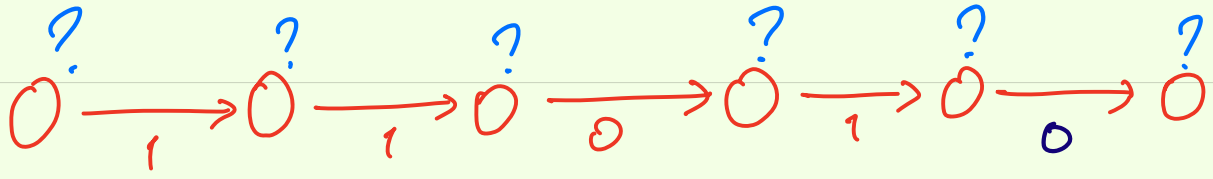


DFA M:



Exercise:

The equiv. DFA on w:



In general, the equivalent DFA $M = (Q', \Sigma, \delta', q_0', F')$ is defined as follows:

- $Q' \subseteq 2^Q$
- $q_0' = \{q_0\}$ (Note: $q_0 \neq \{q_0\}$)
- $F' = \{s \in Q' \mid s \cap F \neq \emptyset\}$
- $\delta'(s, a) = \bigcup_{q \in s} \delta(q, a)$ //