Lecture 5

Example:

1) M: $L(M) = \emptyset$

2) M: $L(M) = \Sigma^*$

3) M: $L(M) = \Sigma^* - \{\lambda\}$

4) M: $L(M) = \{\lambda\}$

Designing Finite Automata

Given a language $L \subseteq \Sigma^*$, design an FA that accepts it.

Example: Design an FA that accepts all the strings containing an even number of 1s. ($\Sigma = \{0,1\}$).

i.e. $L = \{ w \in \Sigma^* \mid |w|_1 = 2k, \ k \in \mathbb{N} \}$

Need to construct an FA $M = (Q, \Sigma, \delta, q_0, F)$ s.t. $L = L(M)$.

? ? ? ? ?

Idea: Count modulo 2.

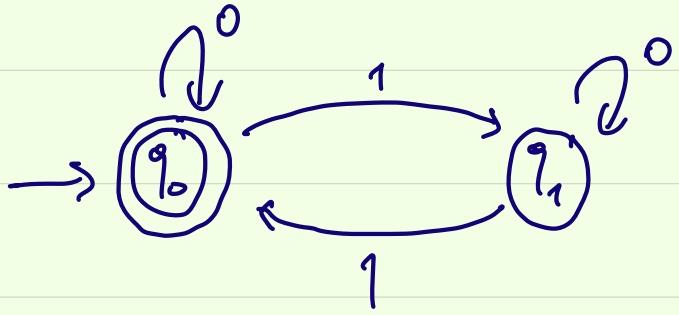
q_0 ... "even number 1's" final, initial

q_1 ... "odd number of 1"

$Q = \{q_0, q_1\}$ $\Sigma = \{0, 1\}$

$F = \{q_0\}$ $\delta = ?$

q_0



$\lambda \in L$

$1 \notin L$

$01101 \notin L$.

0110011

Exercise: FA to accept all strings with odd number of 1s?

Example: FA to accept all strings that have "aba" as a substring.

i.e. $L = \{w \in \Sigma^* \mid w = w'aba\,w'', \; w', w'' \in \Sigma^*\}$

$\lambda \notin L$ $a \notin L$, $b \notin L$, $ab \notin L$, $abbba \notin L$

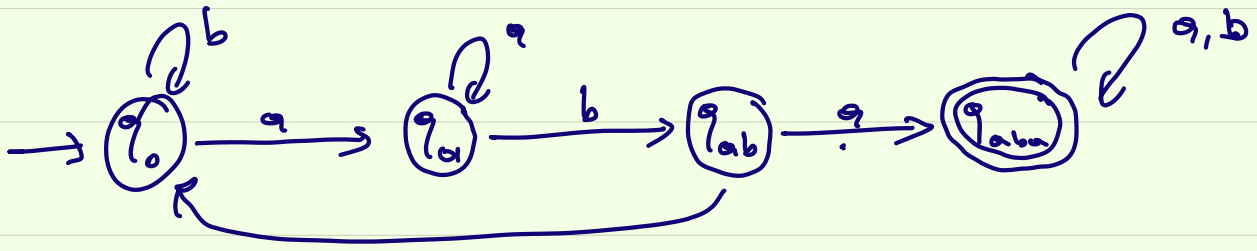
$aba \in L$, $aabab \in L$, ... $abbaba$

q_a ... "an a is seen"

q_{ab} ... "an ab is seen"

final ... q_{aba} ... "an aba is seen."

q_0 ... initial state.



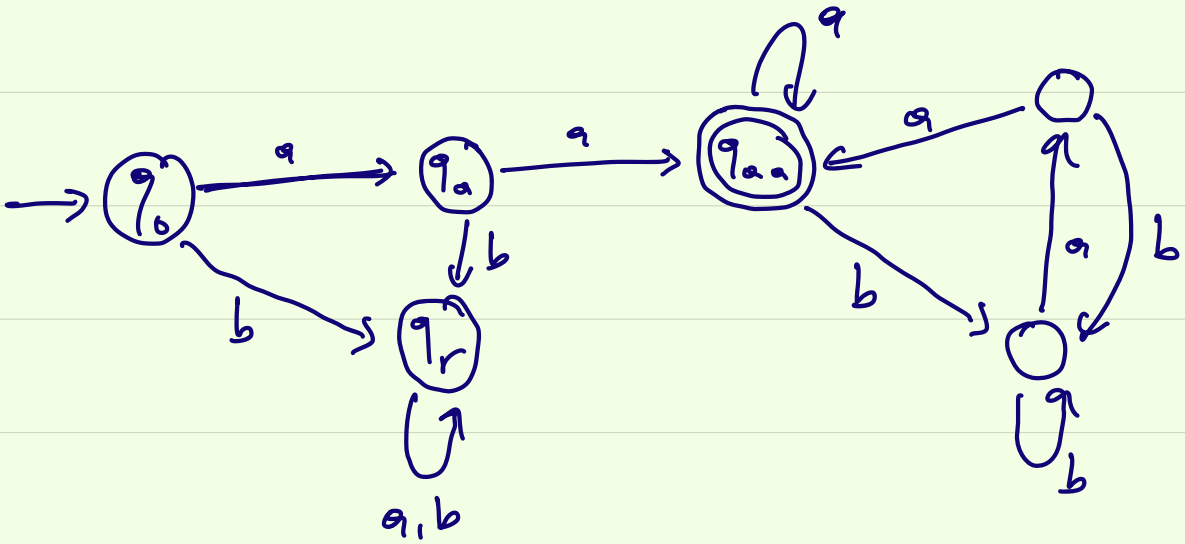
Exercise FA to accept all strings that have "bba" as a substring.

Example: FA to accept all strings that begin and end with "aa".

λ, a, b, bb, bw', w'b, w'ab, X

aa, aaa, aa w'aa s.t. $w' \in \Sigma$ ✓

$L = \{ w \in \Sigma^* \mid w = aa\, w'aa,\ w' \in \Sigma^* \} \cup \{aa, aaa\}$



aabbaa ✓

aababaa ✓

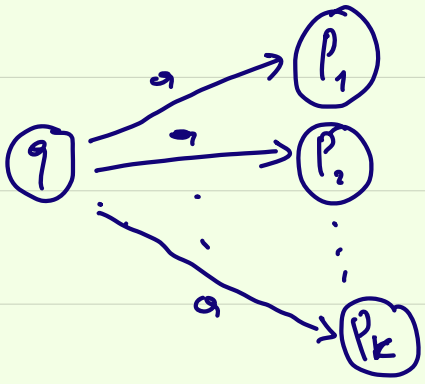
Nondeterministic Finite Automata (NFA)

So far, at any given state q, an FA jumps to the next state deterministically;

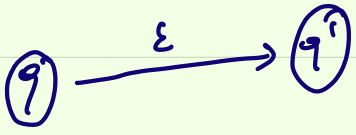
$$\delta(q, a) = q' \quad \text{-- unique.}$$

Hence, we call such FA "deterministic FA" (DFA).

In an NFA, given a state q and an input symbol a, there is a set of possible next states. i.e. $\delta(q, a) = \{p_1, p_2, \ldots, p_k\} \subseteq Q$

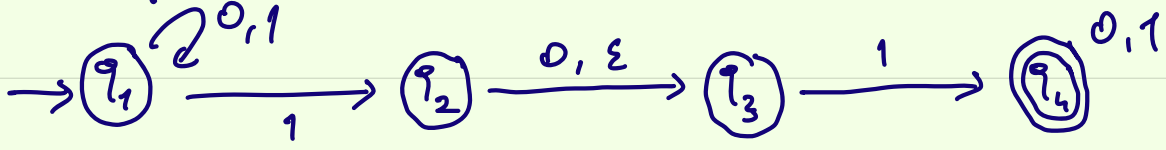


Also, in an NFA, there may be ε-transitions:



That is, N can jump from q to q' without consuming any symbol.

Example:

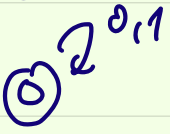
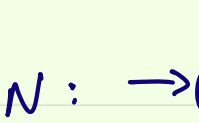


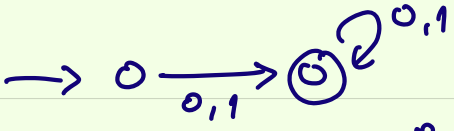
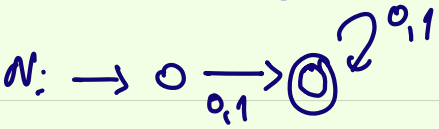
$$\delta(q_1, 1) = \{q_1, q_2\}$$
$$\delta(q_1, 0) = \{q_1\}$$
$$\delta(q_3, 0) = \emptyset$$

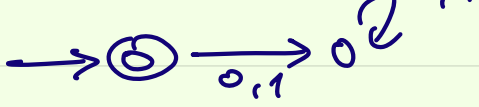
To be cont.

Example:

1) M:  N:

2) M:  N: 

3) M:  N: 

4) M:  N: 