Lecture 15

Regular Grammars

Definition: A regular grammar is a CFG in which each rule is in one of the following forms:

- $A \longrightarrow aB$
- $A \longrightarrow a$
- $A \longrightarrow \varepsilon$

$(A, B \in V, a \in \Sigma)$

Example:

1) $B \longrightarrow aBb \mid Bb \mid b$ ... not regular

2) $S \longrightarrow aB \mid \varepsilon$
$B \longrightarrow bS \mid bA$
$A \longrightarrow aA \mid \varepsilon$
... regular

3) G: $S \longrightarrow aSa \mid aBa$
$B \longrightarrow bB \mid b$
... not regular

Equivalence between Reg. Grammars and FA

Lemma: $L = L(M)$ for some DFA M
$\Rightarrow L = L(G)$ for some reg. gr. G.

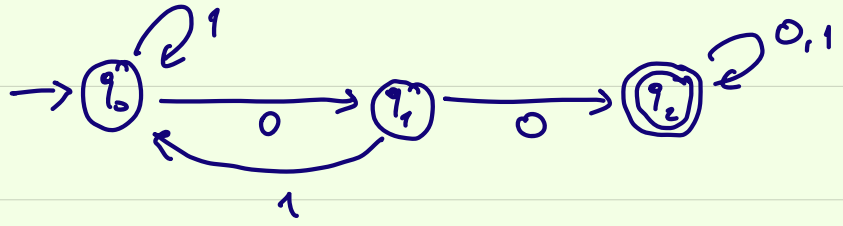
Proof: Given a DFA $M = (Q, \Sigma, \delta, q_0, F)$, we construct a reg. gr. $G = (V, \Sigma, R, S)$ as follows:

- $V = Q$
- Input alphabet Σ is now the set of terminals.
- $S = q_0$
- R contains the following rules
  - For $\delta(q, a) = q'$, add a rule $q \to aq'$
  - For each $q_f \in F$, add a rule $q_f \to \varepsilon$



Rule as transitions
Derivation as computations

Example:

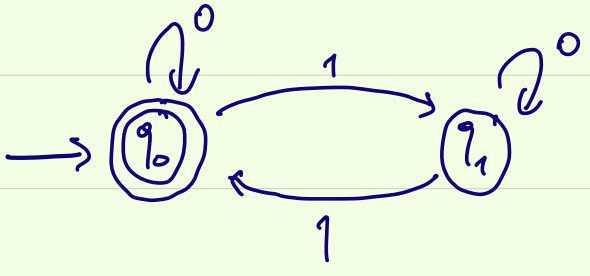


$q_0 \to 1q_0$
$q_0 \to 0q_1$
$q_1 \to 1q_0$
$q_1 \to 0q_2$
$q_2 \to 0q_2$
$q_2 \to 1q_2$
$q_2 \to \varepsilon$

or

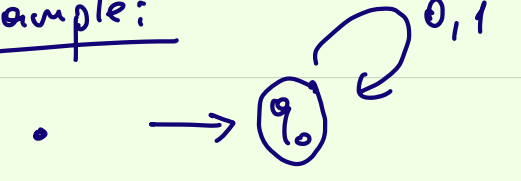
$q_0 \to 1q_0 \mid 0q_1$
$q_1 \to 1q_0 \mid 0q_2$
$q_2 \to 0q_2 \mid 1q_2 \mid \varepsilon$

Example:

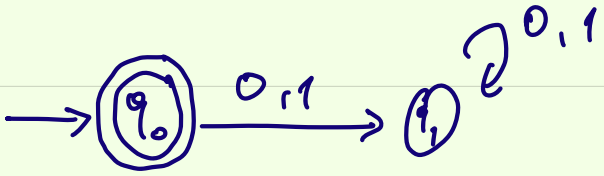


$q_0 \to 0q_0 \mid 1q_1 \mid \varepsilon$
$q_1 \to 0q_1 \mid 1q_0$

Example:



$q_0 \to 0q_0 \mid 1q_0$



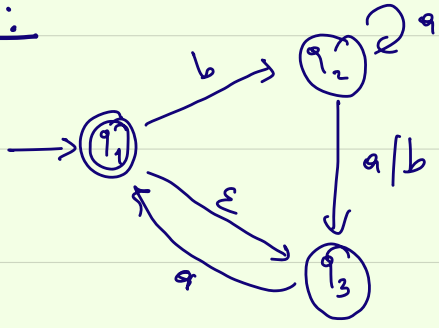
$q_0 \to 0q_1 \mid 1q_1 \mid \varepsilon$

$q_1 \to 0q_1 \mid 1q_1$

Exercise: Reg. gr for 

Remark: The construction applies to NFAs as well.

Example:

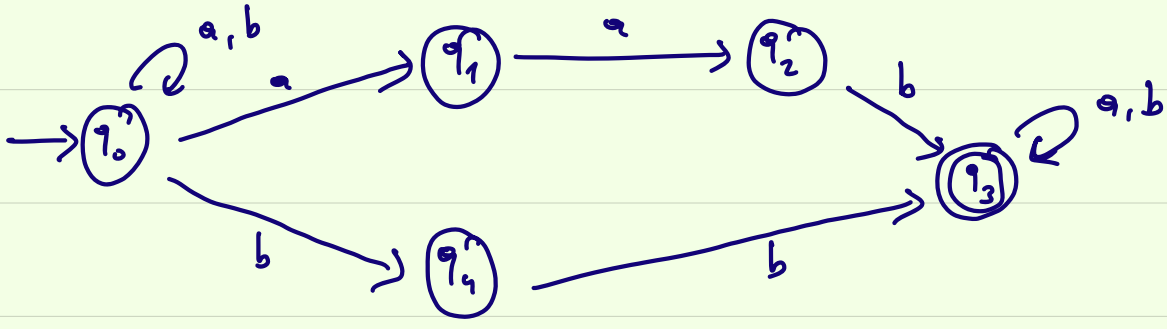


$q_1 \to bq_2 \mid \varepsilon q_3 \mid \varepsilon$

$q_2 \to aq_2 \mid aq_3 \mid bq_3$

$q_3 \to aq_1$

Example:



$q_0 \to aq_0 \mid aq_1 \mid bq_0 \mid bq_4$

$q_1 \to aq_2$

$q_4 \to bq_3$

$q_2 \to bq_3$

$q_3 \to aq_3 \mid bq_3 \mid \varepsilon$

Lemma: $L = L(G)$ for some reg. gr G
$\Rightarrow$ $L = L(M)$ for some DFA M.

Proof: Given a reg. gr $G = (V, \Sigma, R, S)$, we construct a DFA $M = (Q, \Sigma, \delta, q_0, F)$ as follows:

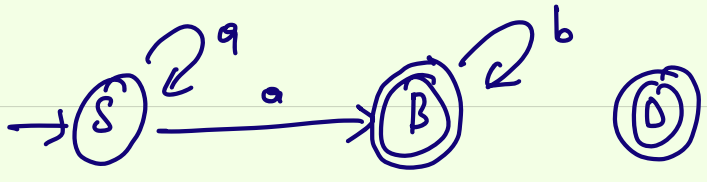
- $Q = V \cup \{D\}$
- $q_0 = S$
- Σ is the set of terminals of G.
- $F = \{A \in V \mid A \to \varepsilon \in R\} \cup \{D\}$
- δ is defined by

$$\delta(A, a) = B \quad \text{iff} \quad A \to aB \in R$$

$$\delta(A, a) = D \quad \text{iff} \quad A \to a \in R$$

Example: $S \to aS \mid aB$

$B \to bB \mid \varepsilon$

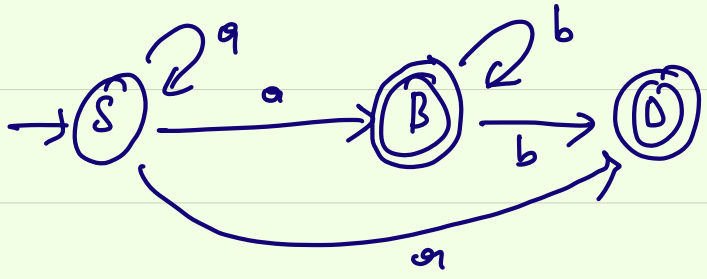


(Now, convert this NFA into an equiv. DFA. left as an exercise.)

Example:

$S \to aS \mid aB \mid a$

$B \to bB \mid \varepsilon \mid b$

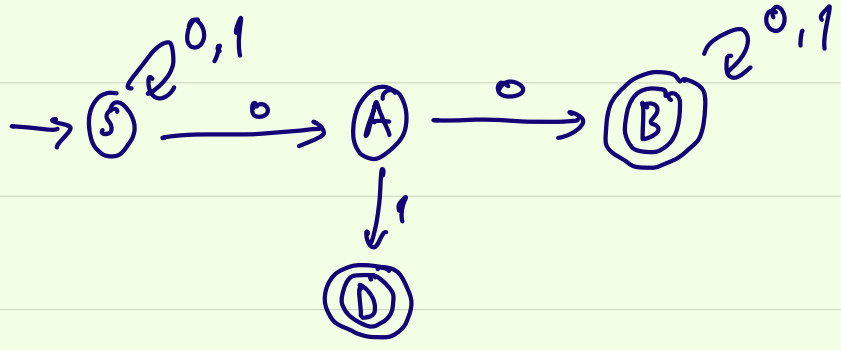


Example:

$S \to 0S \mid 1S \mid 0A$

$A \to 0B \mid 1$

$B \to 0B \mid 1B \mid \varepsilon$



Note: This is an NFA. Convert it to a DFA.