Lecture 13

Example: A CFG that describes the syntax for regular expressions over $\Sigma = \{a, b\}$.

$\Sigma = \{a, b\}$ ... terminals

$V = \{re\}$

$re \longrightarrow \emptyset$

$re \longrightarrow \varepsilon$

$re \longrightarrow a$

$re \longrightarrow b$

$re \longrightarrow re \cdot re$

$re \longrightarrow re + re$

$re \longrightarrow re^*$

e.g. $ab^* + \varepsilon$

$re \Rightarrow re + re$

$\Rightarrow re \cdot re + re$

$\Rightarrow a \cdot re + re$

$\Rightarrow a \cdot re^* + re$

$\Rightarrow a \cdot b^* + re$

$\Rightarrow a \cdot b^* + \varepsilon$

Definition: Let $G = (V, \Sigma, R, S)$ be a CFG

For a rule $A \to \alpha$, and strings $\beta, \gamma \in (\Sigma \cup V)^*$, we write $\beta A \gamma \Rightarrow_G \beta \alpha \gamma$

i.e. $\beta A \gamma$ directly rewrites to $\beta \alpha \gamma$.

- For $\theta_0, \theta_1, \ldots, \theta_n$, $n \geq 0$, $\theta_i \in (\Sigma \cup V)^*$ $i = 0, \ldots, n$, such that

$$\theta_0 \Rightarrow_G \theta_1 \Rightarrow_G \theta_2 \Rightarrow_G \cdots \Rightarrow_G \theta_n$$

we write $\theta_0 \Rightarrow_G^* \theta_n$

Remark:

$A \to \alpha$ context-free

e.g. $aAb \to \alpha$ context-sensitive

e.g. $aBAc \to cA$

Definition: Given a CFG $G = (V, \Sigma, R, S)$

- If $S \Rightarrow^* \alpha$, then α is called a sentential form.
- If $S \Rightarrow^* \alpha$ and $\alpha \in \Sigma^*$, then α is called a sentence.

- The language generated by G is

$$L(G) = \{ w \in \Sigma^* \mid S \Rightarrow^* w \}$$

- Two CFGs G_1 and G_2 are equivalent if $L(G_1) = L(G_2)$.
- A language $L \subseteq \Sigma^*$ is "context-free" if $L = L(G)$ for some CFG G.

Note: If we have rules $A \to \alpha_1, A \to \alpha_2, \ldots, A \to \alpha_n$, then we write

$$A \to \alpha_1 \mid \alpha_2 \mid \ldots \mid \alpha_n$$

Example: G: $S \to aSa \mid aBa$, $B \to bB \mid b$ $\quad L(G) = ?$

$S \Rightarrow^* aba$

$S \not\Rightarrow^* a$

$S \not\Rightarrow^* \varepsilon$

$S \Rightarrow^* ab$

$S \Rightarrow aSa$

$\Rightarrow a^k S a^k$

$\Rightarrow a^{k+1} B a^{k+1}$

$\Rightarrow a^{k+1} b^m B a^{k+1}$

$\Rightarrow a^{k+1} b^{m+1} a^{k+1}$

Note: $k+1 > 0$, $m+1 > 0$

$$L(G) = \{ a^{k+1} b^{m+1} a^{k+1} \mid k+1 \geq 0,\ m+1 > 0 \}$$