Lecture 12

CONTEXT-FREE GRAMMARS (CFG)

Definition: A CFG G is a quadruple

$G = (V, \Sigma, R, S)$ where

- V is a finite set of nonterminals / variables
- Σ is a finite set of terminals / constants
- $R \subseteq V \times (\Sigma \cup V)^*$ is a set of rules of the form $A \to \alpha$ ($A \in V$, $\alpha \in (\Sigma \cup V)^*$)
- S is the initial symbol. ($S \in V$)

Example: G : $S \to 0S1$, $S \to \#$ $\quad V = \{S\}$, $\Sigma = \{0, 1, \#\}$

$L(G)$

e.g.
- $\varepsilon \notin L(G)$
- $1 \notin L(G)$
- $\# \in L(G)$
- $0\#1 \in L(G)$

  $S \Rightarrow 0S1 \Rightarrow 0\#1$
- $000\#11 \notin L(G)$
- $00\#11 \in L(G)$

  $S \Rightarrow 0S1 \Rightarrow 00S11 \Rightarrow 00\#11$

In general $L(G) = \{0^n \#1^n \mid n \geq 0\}$

Exercise: Prove that $L(G)$ is not regular.

Exercise: CFG for $L = \{0^n 1^n \mid n \geq 0\}$

CFG for $L = \{0^n 1^n \mid n \geq 1\}$