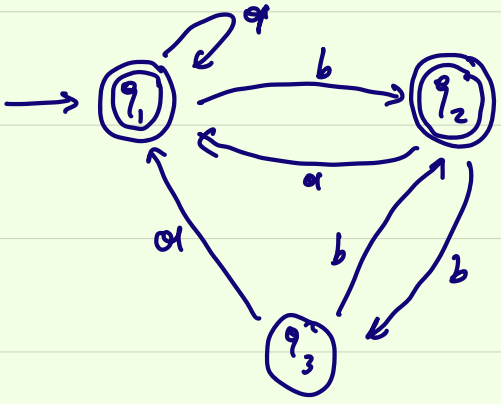
Lecture 10

Example: M:

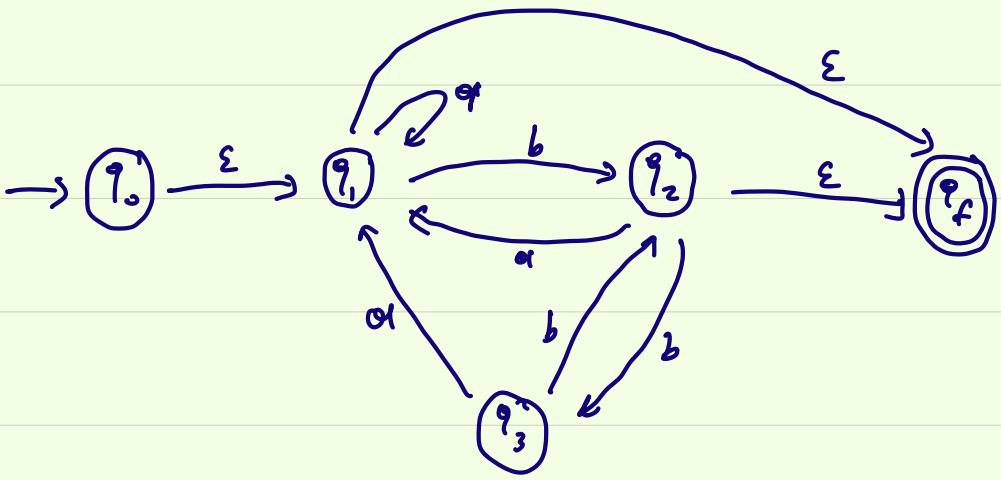


The equiv. RE?

note: $L(M) = L(M')$

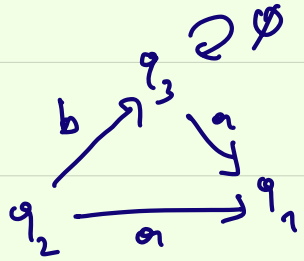
The equiv. GNFA M'

$\Sigma = \{a, b\}$



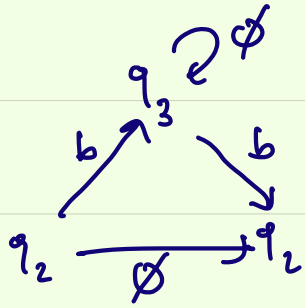
Eliminate q_3:

Recall: $\emptyset^* = \varepsilon$



$\rightsquigarrow$ $q_2 \xrightarrow{b \cdot \emptyset^* \cdot a \,+\, a} q_1$

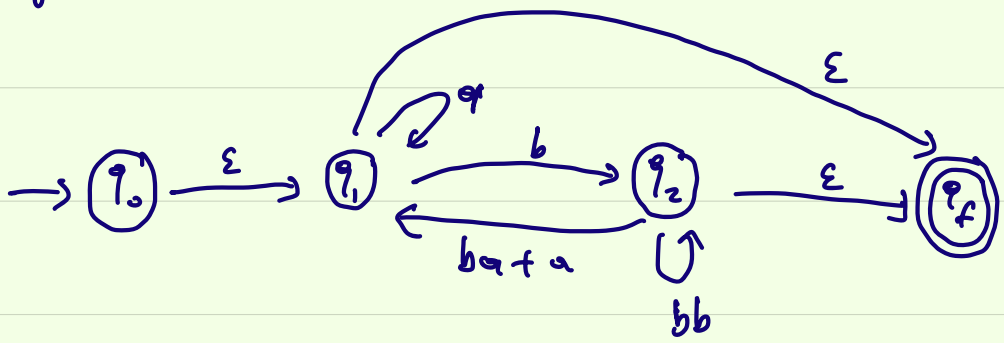
$\rightsquigarrow$ $q_2 \xrightarrow{ba \,+\, a} q_1$



$\rightsquigarrow$ $q_2 \xrightarrow{b \cdot \emptyset^* \cdot b \,+\, \emptyset} q_2$

$\rightsquigarrow$ $q_2 \xrightarrow{b \cdot b} q_2$

We get

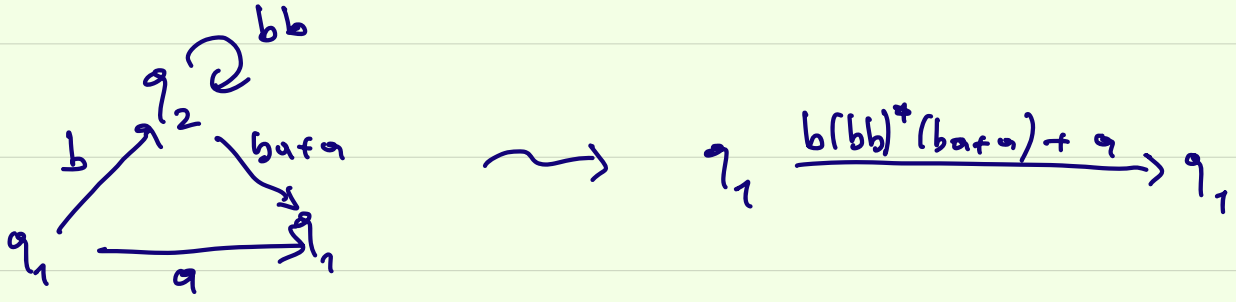


Eliminate q_2:

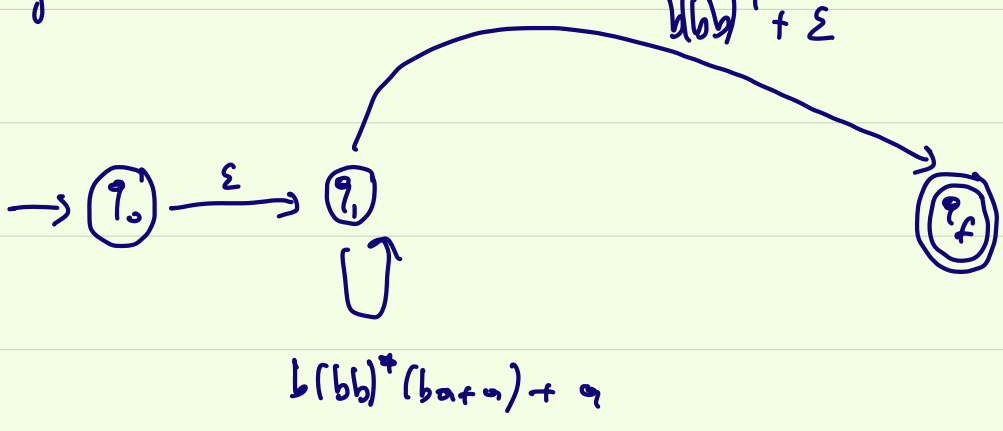
Note: $\alpha + \varepsilon \neq \alpha$

$\alpha + \phi = \alpha$



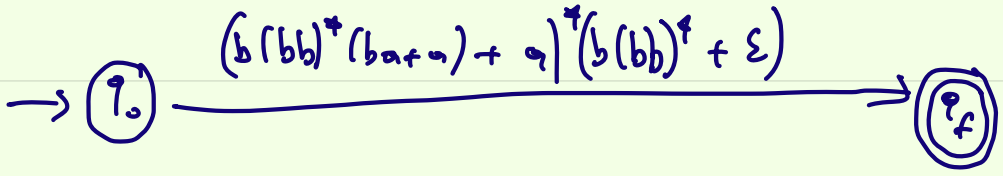


We get



Eliminate q_1:

We get



The equiv. RE is $(b(bb)^*(ba+a)+a)^*(b(bb)^*+\varepsilon)$.

The Pumping Lemma for Regular Languages

Q: How to prove that a language is regular?

A: Construct an NFA or a DFA or a RE that accepts it.

Q: How to prove that a language is not regular?

A: Use the pumping lemma.

Pumping Lemma (P.L.)

$p \Rightarrow q$

p { If a language $L \subseteq \Sigma^*$ is regular, then

q { there exists a positive constant $n > 0$ such that for any string $z \in L$ with $|z| \geq n$, z can be factorized as $z = u \cdot v \cdot w$ satisfying the following properties:

i) $|u \cdot v| \leq n$

ii) $|v| \geq 1$

iii) $\forall i \geq 0$, $u \cdot v^i \cdot w \in L$.

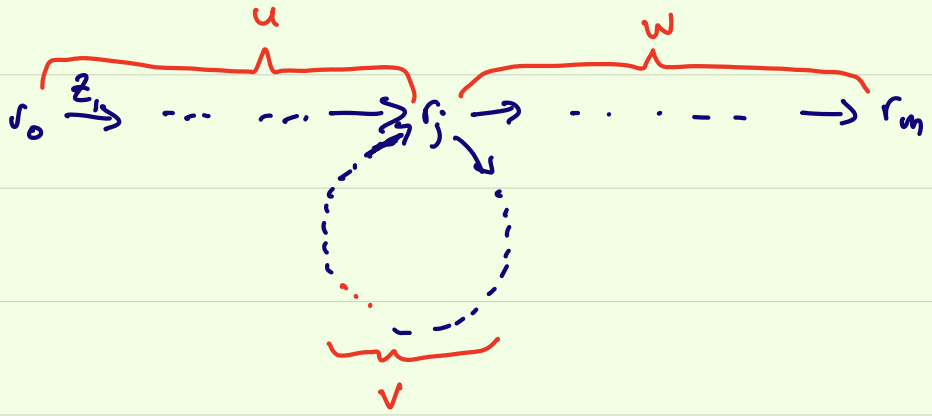
Recall: A computation of a DFA $M = (Q, \Sigma, \delta, q_0, F)$ on a string z has the following form:

$$r_0 \xrightarrow{z_1} r_1 \xrightarrow{z_2} r_2 \rightarrow \cdots\cdots\cdots \xrightarrow{z_m} r_m$$

Suppose there are n states in the DFA M.

i.e. $|Q| = n$.

If $|z| \geq n$, we can redraw the computation above as follows:



Note: P.L. is of the form $p \Rightarrow q$

i.e. it says

"If L is regular, then q holds.

(p)

We know that $p \Rightarrow q$ is logically equivalent to $\neg q \Rightarrow \neg p$

ie. "If q is not true for L, then L is not regular"

How to use Pumping Lemma to prove a language L is not regular:

1) Pick an arbitrary $n \geq 0$.
2) Choose a string $z \in L$ with $|z| \geq n$.

3) Consider a factorization of $t = u \cdot v \cdot w$ and $|u \cdot v| \leq n$ and $|v| \geq 1$.

4) Find an integer $i \geq 0$ s.t. $u \cdot v^i \cdot w \notin L$.

5) Conclude that L is not regular because "q is not true for L".

Example: Show that $L = \{0^k 1^k \mid k \geq 0\}$ is not regular.

Proof:

1) Let $n > 0$.

2) Let $z = 0^n 1^n$. Note $z \in L$ and $|z| \geq n$

3) Consider a factorization of $t = u \cdot v \cdot w$ and $|u \cdot v| \leq n$ and $|v| \geq 1$.

4) Let $i = 0$. and $|v| = \ell \geq 1$

Then $z' = u \cdot v^i \cdot w$ $\quad \ell \leq n$

$= u \cdot v^0 \cdot w$

$= u \cdot \varepsilon \cdot w$

$= u \cdot w$

$= 0^{n-\ell} 1^n$



$\Rightarrow n - \ell \neq n$ since $\ell \geq 1$

$\Rightarrow z' = 0^{n-\ell} 1^n \notin L$.

5) L is not regular because

"q is not true for L".

Example: $L = \{ 1^{k!} \mid k \geq 1 \}$ is not regular.

Proof:

1) Let $n > 0$.

2) Let $z = 1^{n!}$. Then $z \in L$ and $|z| \geq n$.

3) Consider a factorization of $z = u \cdot v \cdot w$

and $|u \cdot v| \leq n$ and $|v| \geq 1$.

4) Let $i = 2$. Then $z' = u \cdot v^2 \cdot w = u \cdot v \cdot v \cdot w$

and let $|v| = l$.

$\Rightarrow z' = 1^{n! + l}$ where $1 \leq l \leq n$

but $n! + l$ is not a factorial.

because $n! < n! + 1 \leq n! + n < (n+1)!$

$\Rightarrow z' \notin L$.

5) L is not regular.