Lecture 1

Math Review

Relations:

Let A and B two sets. The **cartesian product** $A \times B$ is defined as

$$A \times B = \{ (a,b) \mid a \in A \text{ and } b \in B \}$$

e.g. $A = \{1, 2\}$ $B = \{a, b\}$

$$A \times B = \{ (1,a), (1,b), (2,a), (2,b) \}$$

$B \times A = ?$

e.g. $A = \{1, 2\}$ $B = \{a, b, c\}$

$A \times B = ?$

A **relation** R between two sets A and B is a **subset** of $A \times B$.

e.g. $A = \{1, 2\}$ $B = \{a, b\}$

$$A \times B = \{ (1,a), (1,b), (2,a), (2,b) \}$$

"binary relations"

$R_1 = \emptyset \subseteq A \times B$

$R_2 = \{(1,a)\}$ $R_3 = A \times B$

Exercise: $A \times B = B \times A$?

$A \times A = ?$

$A \times A \times A = ?$ $\quad A \times B \times C = ?$

$A \times A \times A \times A = ?$

- Find relations between B and A?
- How many relations are there between A and B?

A function f from a set A to a set B, $f: A \to B$, is a relation with the property that every element of A is mapped to exactly one element of B.

e.g. $A = \{1, 2\}$ $\quad B = \{a, b\}$

$A \times B = \{(1,a), (1,b), (2,a), (2,b)\}$

$R = \phi$ .. not a function

$R = A \times B$ .. "

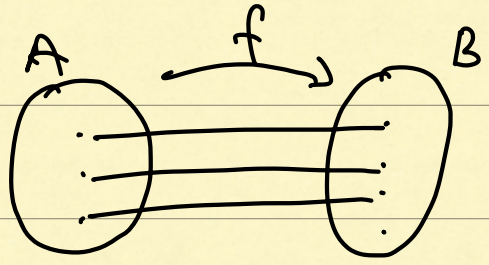
not 1-1 $\quad R = \{(1,b), (2,b)\}$ ✓

1-1 $\quad R = \{(1,b), (2,a)\}$ ✓ onto

Exercise: How many functions from A to B?

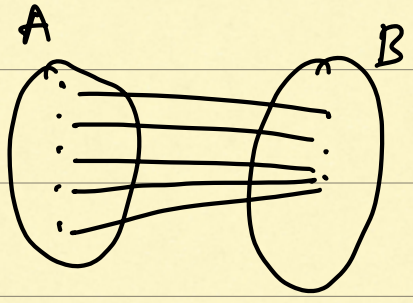
1-1 functions

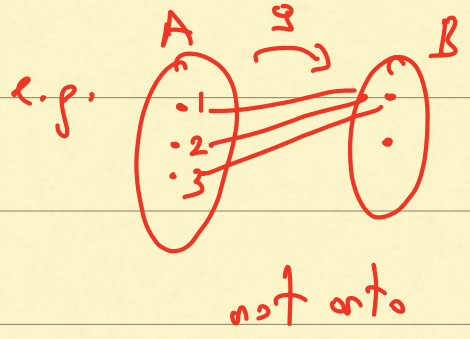
$f: A \to B$ is 1-1 if for every pair $a, b \in A$ we have $a \neq b \implies f(a) \neq f(b)$



Onto functions:

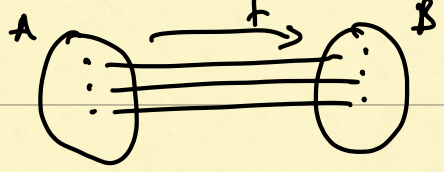
$f: A \to B$ is onto if for every $b \in B$ there exists $a \in A$ s.t. $f(a) = b$.

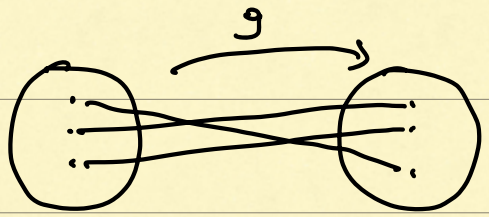




Bijections:

$f: A \to B$ is a bijection if it is both 1-1 and onto.





Popular Sets: $\mathbb{N}$, $\mathbb{Z}$, $\mathbb{Q}$, $\mathbb{R}$

$\mathbb{N}\times\mathbb{N}$, $\mathbb{Z}\times\mathbb{Z}$

$f: A \to B$ ... unary $f(x)$

$f: A \times B \to C$ ... binary $f(x,y)$

$f: A \times B \times C \to D$ ... ternary $f(x,y,z)$

$f: A_1 \times A_2 \times \dots \times A_n \to B$ n-ary