CS 4384 — Automata Theory, Homework 4

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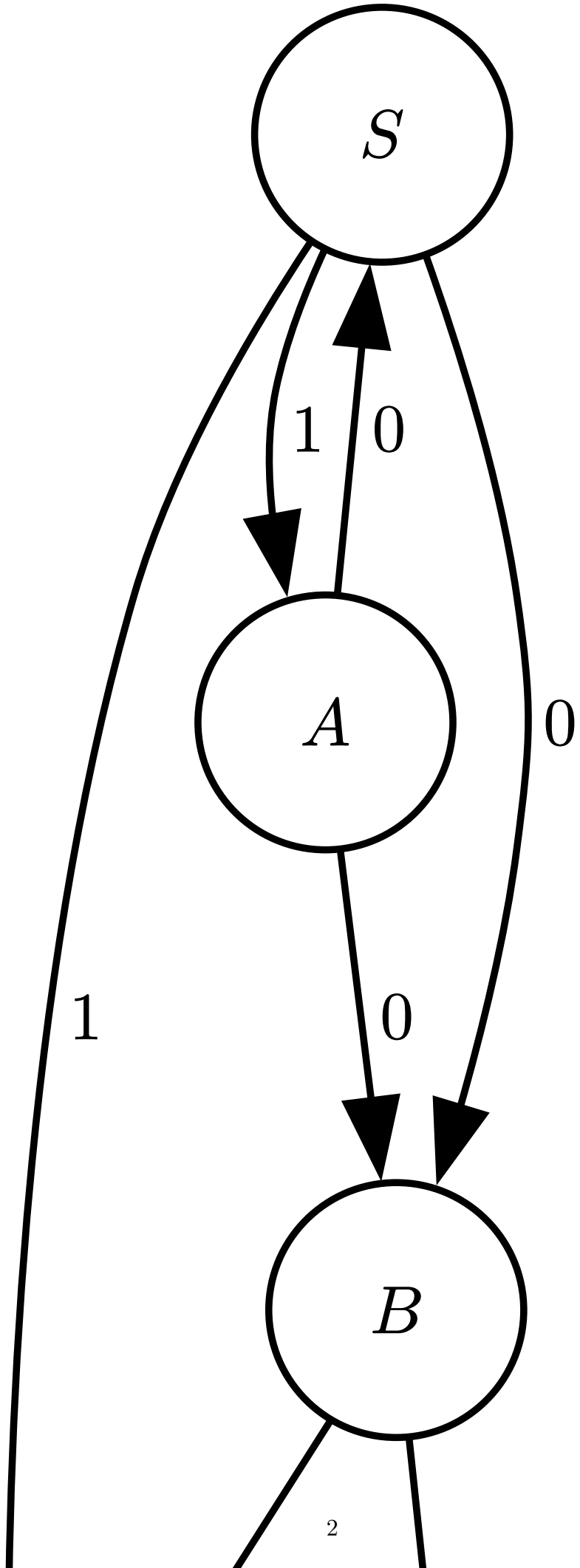
Construct a CFG for the DFA below

$$S \to 0A \mid 1B$$
$$A \to 0A \mid 1C$$
$$B \to 1B \mid 0C$$
$$C \to 0C \mid 1C \mid \varepsilon$$

Convert the following CFG G into an NFA that accepts L(G)

$$S \to 1A \mid 0B \mid 1$$
$$A \to 0S \mid 0B$$
$$B \to bC \mid b$$
$$C \to cC \mid \varepsilon$$

First, we must observe that this is a *regular grammar*. If it was not, then there would be no NFA that accepts L(G) Following the steps to convert a CFG for a regular grammar into an NFA, we can see that the CFG is equivalent to the following NFA:


S
1
0
A
0
1
0
B

Eliminate ε-productions from the grammar below

$$S \to aSa \mid BC$$
$$B \to bbC \mid bb$$
$$C \to cC \mid \varepsilon$$

Here we must eliminate the rule $C \to \varepsilon$. To remove this, we add the following rules:

$$S \to B$$
$$B \to bb$$
$$C \to c$$

In other words, for each rule that involved C, we added a rule substituting C with ε.
This gives the final CFG

$$S \to aSa \mid BC \mid B$$
$$B \to bbC \mid bb$$
$$C \to cC \mid c$$

Eliminate chain productions from the grammar below

$$S \to ABC$$
$$A \to aC \mid D$$
$$B \to bB \mid A \mid \varepsilon$$
$$C \to Cc \mid Ac \mid \varepsilon$$
$$D \to aa$$

We must eliminate the rules

$$A \to D$$
$$B \to A$$

To do this, we simply replace these as follows:

$$A \to aa$$
$$B \to aC \mid D$$

Since we now have the rule $B \to D$, we must apply the same procedure again to get

$$B \to aa$$

Taking these together, we get

$$S \to ABC$$
$$A \to aC \mid aa$$
$$B \to bB \mid aC \mid aa \mid \varepsilon$$
$$C \to Cc \mid Ac \mid \varepsilon$$
$$D \to aa$$

Convert the following grammar to Chomsky Normal Form

$$S \to aBa$$
$$A \to aAS \mid bA \mid \varepsilon$$
$$B \to SBS \mid A \mid bb$$

Next, we must remove all ε-productions
Removing the rule $A \to \varepsilon$ we get

$$S \to aBa$$
$$A \to aAS \mid aS \mid bA \mid b$$
$$B \to SBS \mid \varepsilon \mid bb$$

Removing the rule $B \to \varepsilon$ we get

$$S \to aBa \mid aa$$
$$A \to aAS \mid aS \mid bA \mid b$$

$$B \to SBS \mid SS \mid bb$$

There are no chain productions to remove

$$S \to aBa \mid aa$$
$$A \to aAS \mid aS \mid bA \mid b$$
$$B \to SBS \mid SS \mid bb$$

Finally, we create new variables and rules as follows

$$S \to U_a S_1 \mid U_a U_a$$
$$S_1 \to BU_a$$
$$A \to U_a A_1 \mid U_a S \mid U_b A \mid b$$
$$A_1 \to AS$$
$$B \to SB_1 \mid SS \mid U_b U_b$$
$$B_1 \to BS$$
$$U_a \to a$$
$$U_b \to b$$

Every variable is now of the form $A \to BC$ or $A \to a$, and the CFGs are equivalent, thus the grammar is now in CNF.