CS 4384 — Automata Theory, Homework 3

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Show that the language $L = \{w^n \mid w \in \{a, b\}^*, n \geq 2\}$ is not regular.

We show this by the pumping lemma.

1. Let $n > 0$
2. Let $z = (b(a^n))^n$. Note that $z \in L$ and $|z| \geq n$
3. Consider factorization $t = u \cdot v \cdot w$ and $|u \cdot v| \leq n$ and $|v| \geq 1$
4. Let $i = 0$
   - Observe that $z = b(a^n) \cdot (ba^n)^{n-1}$, by the definition of the power of a string. Further observe that $|u \cdot v|$ will always remove the prefix "b", since $|v| > 0$. For $i = 0$, with $a^k \cdot (b(a^n))^{n-1}$, which is not in the language.
5. Thus the language is not regular

Let $L = \{ww \mid w \in \{a, b\}^*\}$. Show that the language $L' = \{w \in \{a, b\}^* \mid w \notin L\}$ is not regular.

1. Let $n > 0$
2. Let $z = a^n ba^n b$. Note that $z \in L$ and $|z| \geq n$
3. Consider factorization $t = u \cdot v \cdot w$ and $|u \cdot v| \leq n$
   1. Note that $u \cdot v = 0^k$.
4. Let $i = 2$. Then $z' = u \cdot v^2 \cdot w$. With $|v| = \ell$,then $z' = 0^{n+\ell}10^n1$, which is not in the language since $n + \ell \neq n$
5. Therefore, the language L is not regular, since regular languages are closed under complement, which implies that nonregular languages are also closed under complement.

Construct a CFG for $L = \{a^i b^j \mid 2i = 3j + 1\}$ using the recursion method. (That is, you should first define L recursively and extract the CFG rules from it.)

The smallest string in the language is clearly *aab*, then *aaaaabbb*. Continuing, we see that the number of *a*s increases 3 at a time, and the number of *b*s two at a time.

1. $G : (\{S\}, \{a, b\}, \{S \to aab \mid aaaSbb\}, \{S\})$

Let $\Sigma = \{1, \#\}$

$$Y = \{w \mid w = x_1\#x_2\#...\#x_k \text{ for } k \geq 0 \text{ , each } x_i \in 1^* \text{ , and } x_i \neq x_j \text{ for } i \neq j\}$$

Show that Y is not regular: Hint(You can use the pumping lemma)

1. Let $n > 0$
2. Let $z = 1^n\#1^{n-1}\#...\#1^1\#1^0$, where $x_i = 1^i$. Note that $z \in L$ and $|z| \geq n$
3. Consider factorization $t = u \cdot v \cdot w$ and $|u \cdot v| \leq n$
4. Let $i = 0$. $u \cdot v$ is contained within the x_1 term. + Thus, the removal of some nonzero amount of characters characters from x_1 is no longer unique, which violates that $i \neq j$, since 1^{n-k} is an x_i within the string for some i.
5. Thus the language is not regular.